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# Spectrum analysis of various modulation schemes to determine the most desirable modulation method for use in tracking of high-altitude satellites.

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# SPECTRUM ANALYSIS OF VARIOUS MODULATION SCHEMES TO DETERMINE THE MOST DESIRABLE MODULATION METHOD FOR USE IN TRACKING OF HIGH-ALTITUDE SATELLITES

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by

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Submitted in partial fulfillment of
the requirements for the degree of

MASTER OF SCIENCE
IN
ENGINEERING ELECTRONICS

United States Naval Postgraduate School
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ABSTRACT

The existence of the space age has put strenuous demands on the communication techniques used in obtaining high-accuracy range, range-rate, angle and angle-rate data to be used in tracking of high-altitude satellites and other space vehicles. The tremendous distances involved require that sophisticated modulation techniques be used to help obtain reliable signals as free from noise as possible.

The important factors which must be considered in picking a modulation method are discussed and a spectrum analysis and a signal-to-noise ratio comparison of two of the more promising methods of modulation (amplitude modulation -single side band- and phase modulation) are made. Through the use of a detail spectrum analysis of different modulation techniques one can see certain advantages and disadvantages of the techniques proposed.

The writer would like to thank Professor George M. Hahn of the U. S. Naval Postgraduate School for the assistance and advice given by him during the preparation of this analysis.

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## 1. Introduction

Before the era of the guided missile, satellites, and other space vehicles, communications was concerned mainly with transferring information or data from one point on earth to another. Space has brought a new dimension into the picture, and the distances between communicating bodies has increased enormously.

The information used by an engineer in tracking a space craft is range, range rate, direction cosines, and angle rates. With the great distances involved, the accuracy required of this information is exacting and requires the most sophisticated techniques in communication to approach the accuracies desired.

A brief review of the basic principles of radio tracking is made so that an approach to the signaling problem can be made with a better understanding of what data is required, and what problems are associated with the sending of that data.

Secondly, the choice of the form of signal being sent is analyzed, and the reasons for narrowing down the form of modulating the signal to either amplitude modulated--single side band or phase modulated are discussed.

A closer examination of the two forms of modulating the desired signal is made and a spectrum analysis and signal-to-noise ratio comparisons are made.

Interpretation of the results in the third step is made and recommendations based on the research above are presented.

2. A Brief Review of the Basic Principles of Radio Tracking.

A. Data Required. Though all are not necessary at once, there are basically four types of information used in tracking a satellite or a space vehicle. They are: range, range rate, angle and angle rate.

B. Basic scientific principles applied to obtain range, range rate, angle and angle rate.

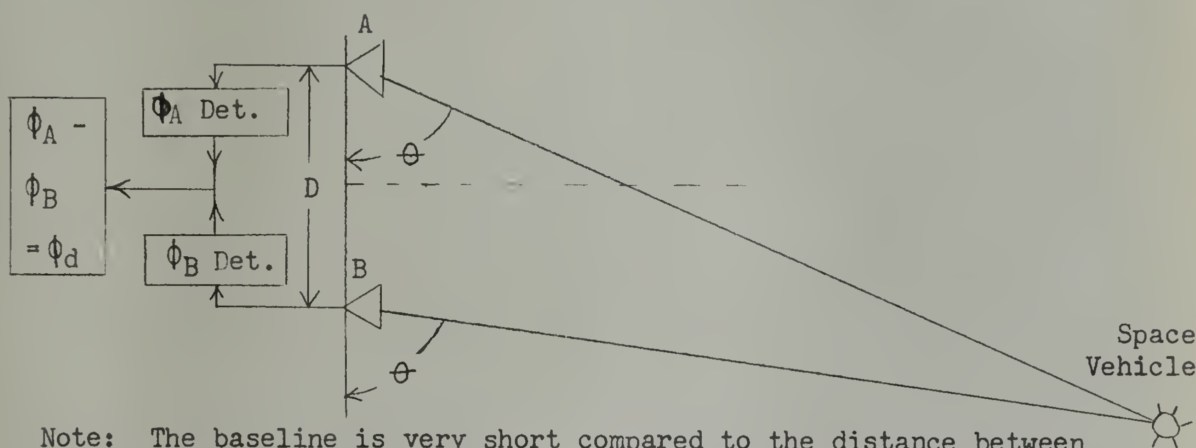
Three basic scientific principles are used to help obtain the information required in tracking a space vehicle. The first principle is that electromagnetic waves travel with a finite and known speed.

The second principle is that the instantaneous phase of a traveling wave (with respect to a standard reference) changes linearly with the distance traveled. This is a very powerful tool and can be used to obtain both range and direction cosines.

To obtain range using the second principle, a signal is generated and part of it is sent toward the space vehicle, and part of it is stored and its phase is recorded. The signal reaches the missile and is received by a transponder which then retransmits the signal back to the tracking station. Upon the return of the signal, the tracking station equipment subtracts the effect of the velocity of the space vehicle (doppler effect), and any phase shift due to the circuitry of the transponder and the tracking station. It then compares the phase of the received signal to the phase of the signal when it was initially sent. The difference in phase angle is linearly related to the total distance to and from the satellite and hence range can be obtained.

Similar to the way our ears locate the position of a sound, the direction cosines can be obtained from one tracking station by again

using the principle of linear phase shift with distance traveled of the signal. Effectively, two antennas are separated by a finite distance and then are placed parallel to each other and perpendicular to their base lines. The phase difference of the received signal by each antenna is proportional to the direction cosines. A brief illustration of the method of obtaining the direction cosine is shown in figure 1 below:[1]




Note: The baseline is very short compared to the distance between the space vehicle and the transmitting station, and the signal is effectively a plane wave normal to a line between the station and vehicle.

$$\cos \theta = \frac{\phi_d C}{\omega_c D}$$

$\phi_d$ = phase difference between received signals (in radians)

C = speed of light

$W_c$ = frequency of signal sent (radians/second)

D = baseline length (in KM)

Figure 1.

[1]Adcom Inc., First Quarterly Report on High-Accuracy Satellite Tracking Systems, Adcom Inc., p. 7, June 30, 1961.

Acquirement of velocity information can be obtained by applying the third basic principle--the doppler effect. The frequency, $f_{rec}$, received by an observer moving with velocity $V_{obs}$ towards a source which in turn is approaching the observer with velocity $v_{source}$ and which is emitting a note of frequency $f_{trans}$ is given by:[2]

$$f_{rec} = \left(\frac{C - V_{OBS}}{C - V_{source}}\right) f_{trans}$$

c = velocity of wave propagation in the intervening medium

$$= \frac{1}{\sqrt{\mu \epsilon}}$$

C. Tracking Systems.

The three basic types of tracking systems categorized on the basis of where the measurement of range, velocity and angle measurement are made are:

a. ground-centralized

b. vehicle-centralized

c. bi- or multi-central

In a ground-centralized system the major transmitting and receiving equipment are located at some point on earth, and the data is recorded and analyzed on the ground. The vehicle may or may not have a transponder. If the vehicle does bear a transponder the ground-centralized system is known as being active; and likewise if there is no transponder, then the system is called passive.

[2]Ibid., p. 4.

Figure 2 is a block diagram of a basic ground-centralized system.[3]

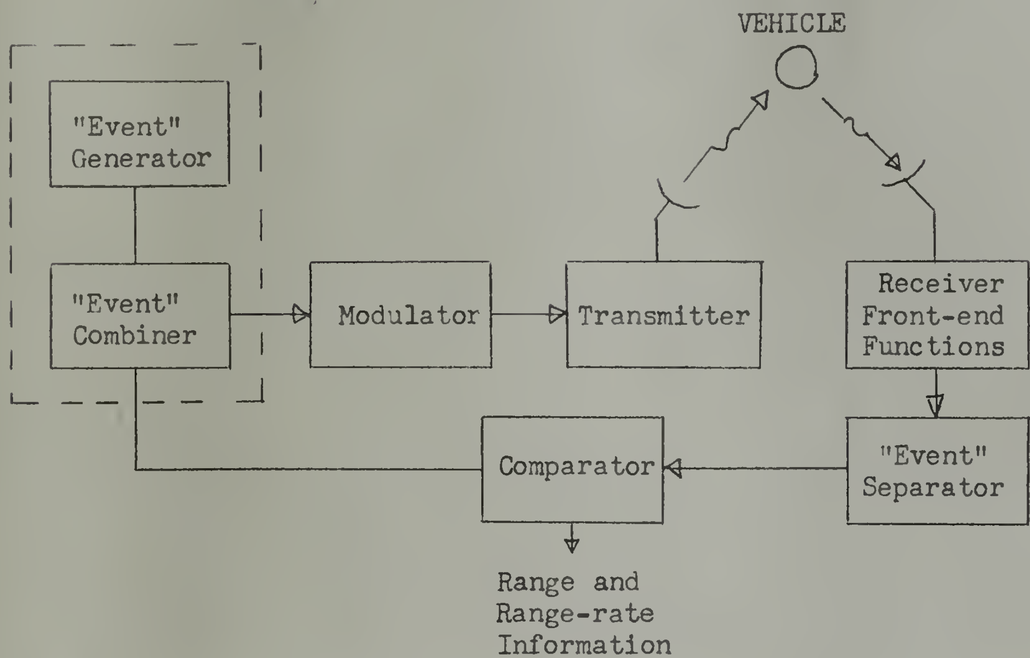



Figure 2. Basic ground-centralized system.

A vehicle-centralized tracking system has the transmitting and receiving equipment located within the vehicle. Range and range-rate information is obtained again by comparing the received signal against the transmittal signal. Similar to ground-centralized systems the vehicle-centralized system is called active, if there is a station located on earth which receives the transmitted signal, amplifies it, and retransmits it back to the vehicle. A lack of such a station classifies the system as passive. An interesting example of a passive vehicle-centralized system will be a manned rocket ship orbiting or

[3]Adcom Inc., Second Quarterly Report on High-Accuracy Satellite Tracking Systems, Adcom Inc., pp. 4-8, Sept. 30, 1961.

approaching a planet which lacks any form of life and which is void of a cooperating transponder.

The multi-central tracking system is really a combination of the other two systems, and range and range-rate information is accumulated independently by both the vehicle and the ground station. This system has the advantage of redundancy and the information can be correlated for more accurate readings.

At present, most of the space tracking stations come under the ground-centralized system category. In the future when greater pay loads can be launched, and the addition of complex tracking equipment does not limit the mission of the space vehicle, use will be made of vehicle- or multi central-tracking systems. An advantage of vehicle-centralized systems is that the vehicle can pinpoint its location in space in real-time and relay this information to an interested ground station which is within radio sight of the vehicle. This can be accomplished much faster than by several ground stations making independent readings and then combining the information to triangulate the position of the vehicle.

D. Problems associated with radio tracking of space vehicles.

There are many problems associated with the radio tracking of a space vehicle. Some questions which are always present are whether or not to use a transponder on the vehicle, what accuracy is required, what distances are involved in the space shot, what kind of signal should be sent and how it should be modulated, and should total or incremental tracking be used. Inherent problems in space communication such as noise, uncertainties of the propagating medium, circuit reliability, and the uncertainty in the speed of light must be contended with.

The use of a transponder in a space vehicle is desirable since it will increase the range capabilities of the ground-centralized system. Through its use, the ground based tracking station does not have to have the great transmitting power capabilities of stations which operate without transponders and therefore is less complicated, and hence less costly. The question, therefore, that will have to be considered is whether the increased weight of the space vehicle outweighs the benefits of increased flexibility and reliability of ground reception.

Since radio waves must travel through a medium, i.e. space, it is important to understand the characteristics of this medium, and its effect on the propagation of the radio waves to a space vehicle.

Space, as far as earth tracking of satellites is considered, is an ever varying structure whose foundation is the earth's troposphere. It has the ionosphere with its D, E, $F_1$ and $F_2$ layers, which are ever changing from hour to hour, day to day, year to year. There is the atmosphere with its water vapor and oxygen molecules, and there is the electromagnetic Van Allen Belt which is to be considered in far reaching communications.

The atmosphere can affect the accurate reading of range rate; and the atmosphere's random fluctuations, resulting from high turbulence and inhomogeneities in the atmosphere, make the errors introduced by the atmosphere in recording of range rate unpredictable.

The atmosphere is also responsible for what can be called "refraction anomalies". There are major refraction anomalies due to the existence of water vapor, which essentially exists in the first 10,000 feet of air about the earth's surface. In addition to refraction this

varying water vapor also causes different attenuation to the radio signals for different frequencies.

Another effect for which compensation must be made is the doppler shift in frequency of the radio signal due to the standard effects of the atmosphere and of the ionosphere.

A study of the complete effects on radio propagation due to the medium of space has filled many books and will not be expounded upon further. It is sufficient to remember that many of the errors introduced in a signal which is propagating in the atmosphere can be predicted on a statistical basis, if the condition of the atmosphere (e.g. density of electrons in the ionosphere and variation of the dielectric constant in the troposphere) is known at the time of transmission and can be corrected for. However, it is very difficult to accurately predict the true condition along the path traveled by the radio wave.

One constant source of difficulty in any communication system is noise. Noise is defined as any spurious or undesired disturbances that tend to obscure or mask the signal to be received.[4] In communicating long distances, as we might be concerned with in moon or solar shots, the signal voltage received will be inversely proportional to the distance traveled; while receiver noise is independent of position and remains at a constant level.

Noise is generally broken up into two major classifications--man-made noise and non-man-made noise. A list of a few of the thousand

[4] M. Schwartz, Information, Transmission, Modulation, and Noise, McGraw-Hill Book Company, Inc., York, Pa., p. 197, 1959.

forms of man-made noises are listed below:

(1) Electromagnetic radiation from electrical sources (razors, fluorescent bulbs, neon lights, electrical appliances).

(2) Reception of an undesired signal whose undesirability stems from the fact that the signal (usually one that in its own right carries intelligence) is not wanted.

(3) Pickup through the power supply.

(4) Microphonics--the conversion of mechanical vibration into electromagnetic radiation.

Man-made noises can be very troublesome in space communication. The fact that the noise is often erratic and completely unpredictable makes any communication system vulnerable to its presence. Through proper shielding, filtering, design, and by the elimination of many of the noise sources, some of the man-made noises can be cancelled and many others minimized. However, due to its unpredictability it can never be completely eliminated.

The other source of noise (non-man-made noise) is usually broken up into the following headings:

(1) Atmospheric Noise and Interstellar Interference. Atmospheric noise has bothered any person who has ever tried to listen to an AM broadcast during a lightning storm. Interstellar interference is more sophisticated and is the result of such celestial happenings as two galaxies colliding together. This noise level is quite low, but in future space shots where 250,000 miles is just the first leg of a long journey, it will become more and more important.

(2) Thermal Noise.

At any temperature above $0^{\circ}$ Kelvin there is random movement of free electrons in a material. The noise is a function of the temperature and has a gaussian distribution. In the frequency domain the thermal noise has a band limited uniform spectrum. Communication systems are plagued by thermal noise, and this is usually the limiting factor in the design of a sensitive receiver. Modern parametric amplifiers and super-cooled front end receivers are bringing this noise level down, however, to where other forms of noise are beginning to have a limiting effect.

(3) Tube Noise.

(a) Shot noise. The electron stream bombarding the plate of a tube is really random in nature and this randomness causes the effect known as shot noise.

(b) Partition noise. In the multi-grid tubes such as a tetrode or a pentode there is random electron flow distribution to the different grids and this effect causes partition noise.

(4) Current Noise.

The conductivity of solid state elements is usually a function of the current in the device and can cause a noise whose level is over that specified by thermal noise.

In studying the various forms of modulating the tracking signal a more detailed look at thermal noise will be covered in section 3.

If everything in a radio tracking system were working perfectly, there would be a limit to the range accuracy. This limit is set by the lack of knowledge of the exact speed of light. The most refined measurement available on the speed of light is

$$C = 299792.5 \pm .4 \text{ Km/sec}$$

$\pm \Delta R$ due to $\pm .4$ Km/sec is

$$\pm \Delta R = \frac{R\,(0.4)\text{ KM/SEC}}{299792.5\text{ KM/SEC}} = 1.33 \times 10^{-6}\,R$$

For a shot to the moon, therefore, the best range reading would be within

$$\pm \Delta R = 250{,}000 \text{ miles} \times 1.33 \times 10^{-6} = \pm .333 \text{ miles}$$

and for a shot to the sun, the best range resolution would be within

$$\pm \Delta R = 93{,}004{,}000 \text{ miles} \times 1.33 \times 10^{-6} = \pm 123.8 \text{ miles}$$

The tracking of a space vehicle can be performed either totally, or in increments, or a combination of the two (called "mixed" tracking). Total tracking of a space vehicle means that each set of data received determines the exact position of the vehicle independent of previously acquired data. If the space vehicle is erratic or jumpy in its movements and makes large random movements in range between range measurements, total tracking is almost mandatory. Total tracking is also beneficial if one tracking station is tracking more than one space vehicle at the same time, and it is necessary to obtain tracking data on one vehicle, then switch to the other vehicle, and so forth.

If the space vehicle is making a smooth trajectory whose path can be fairly well predicted, then it is only necessary to obtain difference data. Once the position of the vehicle has been obtained, the position is determined on incrementally gathered data. A computer will plot the expected path of the vehicle, and the received data will be used for

minor corrections. In a way this system acts as a closed loop servo where the received data is the error signal.

A mixed system is basically an incremental system with the addition of total tracking capabilities, whose function is to eliminate ambiguities in range and maintain an accurate reference for the incremental data.

The advantage of the incremental or mixed system over the total tracking system is that they are much more efficient in that they require less data on which to operate, and therefore, they require less channel capacity. To show this, a comparison between channel capacities for an incremental, versus a mixed, versus a total tracking system for the below conditions will be made.[5]

The space vehicle to be tracked will be a lunar probe:

Range: zero to 250,000 miles

Range rate: never exceeding 10 miles/sec

Range resolution: within a 10 foot interval

Measurement rate: 10 times per second

Ambiguity resolution in the mixed system: once every second.

Calculation of Channel Capacity for a Total Tracking System

(1) m = number of 10 foot intervals in 250,000 miles = 250,000 X 528 = 132,000,000 symbols

(2) Entropy[6] $= H(x) = -\sum_{i=1}^{m} \frac{1}{m} \log_2 \frac{1}{m} = \log_2 m$

$$H(x) = \log_2 (132{,}000{,}000) = 26.98 \text{ bits/symbols}$$

[5]Adcom Inc., First Quarterly Report, op.cit., pp. 20-22.

[6]J. C. Hancock, An Introduction to the Principles of Communication Theory, McGraw-Hill Book Co. Inc., York, Pa., p. 157, 1961.

(3) symbol rate = 10 symbols/second

(4) Channel capacity = Entropy X symbol rate
= 26.98 X 10 = 269.8 bits/second

Calculation of Channel Capacity for an Incremental Tracking System

(1) Within 1/10 of a second the position of the missile can only change by $\pm$ 1 mile.

(2) m = number of 10 foot intervals in 1 mile = 1 X 528 = 528 symbols

(3) $H(x) = \log_2 528 = 9.04$ bits/symbol

(4) symbol rate = 10 symbols/second

(5) Channel Capacity = 9.04 X 10 = 90.4 bits/second

Calculation of Channel Capacity for Mixed Tracking System

(1) Channel Capacity for the incremental tracking (see above) = 90.4 bits/second

(2) Total tracking once per second

(a) m = 132,000,000 symbols (see above)

(b) H (x) = 26.98 (see above)

(c) symbol rate = 1 symbol/sec

(d) Channel Capacity = 1 X 26.98 bits/second = 26.98 bits/sec

(3) Total Channel Capacity = 90.4 + 26.98 = 117.38 bits/second

Though incremental tracking is highly efficient, it is much easier to design and build a total tracking system. The use of harmonic signalling which will be discussed later lends itself perfectly to total tracking. It is possible to increase the efficiency of the total tracking system by decreasing the amplitude of the lower tones, though it will not be as efficient as the incremental tracking system.

There are other considerations to think about in the design and

sending of a signal, such as weighing the cost and complexity of equipment versus the accuracy desired in the tracking system. The next section will discuss the signal itself and in what form it could be sent.

3. The Radio Tracking Signal.

The most important single parameter in a radio tracking system is the ultimate signal that is transmitted to and from the space vehicle. The two basic decisions whose results determine the final shape of the transmitted signal are:

(1) Selecting the event or events that will be transmitted. The frequency spectrum of the modulating signal will be chosen so that it can meet its measurement requirements.

(2) The method of modulating the carrier will be decided, weighing all the requirements of accuracy and reliability.

A. <u>The Modulating Signal</u>

The choice of events is determined specifically by the range measurement requirements. The first decision to make is to determine whether to use a total tracking system, mixed-tracking system, or an incremental tracking system. A modified form of total tracking system will be used (i.e. the signal will be continuous and at any one time the vehicle can be located independently of previous signals. Under normal operations only a portion of the signal will be used; and incremental tracking will be in effect, unless it is necessary to resolve ambiguities, which can be done by using the total signal sent). The second decision is to determine what signal will provide the information required. Two types of events which can provide the necessary information are harmonic events and pseudo-random events.

Harmonic events consist of finite number of discrete tones which in our case are harmonics of the same fundamental. The lowest frequency tone (fundamental) chosen is 32 cycles.

Its half-wavelength

$$\text{is} = \frac{C}{2\ f} = \frac{3 \times 10^8}{2(32)} = 4.69 \times 10^6 \text{ meters}$$

$$= 4690 \times .62137 = 2910 \text{ miles}$$

This tone can be chosen to fix the space vehicle in any 2,910 mile interval. Five other tones have been chosen, each being five times the one preceding it. Therefore, the six tones are 32 cps, 160 cps, 800 cps, 4 KC, 20 KC, and 100 KC.

The accuracy of the phase measurement is increased with the frequency of the tone measured. The 100 KC tone will be used for incremental tracking and this will provide accurate range measurement.

If the 100 KC tone is lost or obscured by noise, or if the specific position of the vehicle is in question, then the next tone (20 KC) is used to resolve the ambiguity. If this is not sufficient to resolve the ambiguity, a lower tone is used, and so forth until the space vehicle is again accurately located.

The tones can be of the same amplitude or they can be weighted in accordance with their use and importance. One system would be to weigh the 100 KC tone heavily and lessen the amplitude of the other five tones.

The other form of event is the pseudo-random event. This signal can be of any form or random shape. It can be noise, or pulses, or any complicated waveform whose shape is very difficult to predict. A copy of the signal which is transmitted is kept at the receiving station so that it can be compared with the received signal. The use of this event is greatly beneficial when it is important that the signal be jam proof,

that it be undetectable, or that its design is to combat multiplicative and/or additive noise.

B. Modulation Technique

Once the event has been chosen, it must be determined how this event should modulate the carrier. In this determination, careful thought should be given to the following important points:[7]

(1) Efficient utilization of the transponder power in conveying the tones: since the transponder reduces the productive payload of the space vehicle, the available equipment is quite limited and must be used as efficiently as possible.

(2) The total system should be kept as simple as possible and still perform its function. The more complicated a system becomes, the greater becomes the chance for error.

(3) The total bandwidth should be kept as narrow as possible commensurate with the signal design. Increase in bandwidth means an increase in noise bandwidth, a greater dependency on wideband transmission filters, and hence a greater chance for phase distortion, and finally a decreasing channel capacity efficiency.

(4) Peak factor of the signal: (i.e. ratio of peak value to rms value). The overall effectiveness of a system is based on its average signal power handling capacity. The design of the system is limited by its peak value handling capacity. Therefore, the ideal system would have a peak factor of one.

(5) Demodulation of noise: different forms of demodulation result in different noise average power output, even though the noise spectral density is identical at the input of the demodulator. For instance, in

[7]Adcom Inc., First Quarterly Report, op. cit., pp. 11-13.

amplitude modulation the demodulator is sensitive to the amplitude of the noise components and insensitive to the phase of the noise. While in frequency demodulation, the reverse is true.

Keeping these five important considerations in mind, two general basic forms of modulation (linear and exponential) will be reviewed in a qualitative manner. The modulation techniques, which obviously do not meet the basic considerations, will be eliminated from further review.

The three modulation processes which come under the classification of linear modulation are:

(1) Amplitude Modulation (AM)

(2) Amplitude Modulation-Double Side Band (AM-DSB)

(3) Amplitude Modulation-Single Side Band (AM-SSB)

A quick look at the mathematics of AM will give a review of its basic characteristics and limitations.

AM

Carrier: $f_c(t) = E_c \cos \omega_c t$

Modulating signal: $f_m(t) = E_m \cos \omega_m t$

Amplitude Modulated carrier: $s(t) = K\left[1 + m\, f(t)\right] \cos \omega_c t$

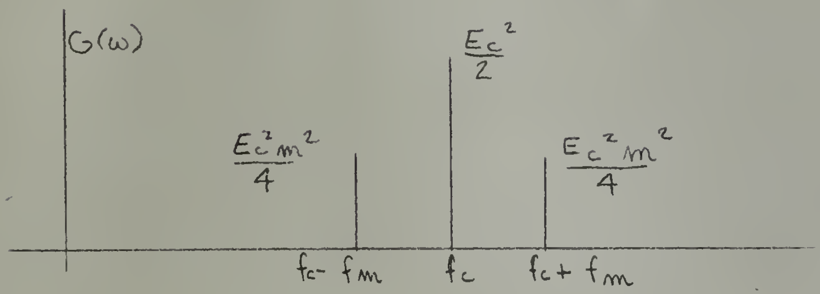
$$s(t) = E_c\left[1 + E_m/E_c \cos \omega_m t\right] \cos \omega_c t$$

$$m = E_m/E_c \leq 1$$

$$s(t) = E_c\left[1 + m \cos \omega_m t\right] \cos \omega_c t$$

$$= E_c \cos \omega_c t + \frac{E_c m}{2} \cos(\omega_c + \omega_m)t + \frac{E_c m}{2} \cos(\omega_c - \omega_m)t$$

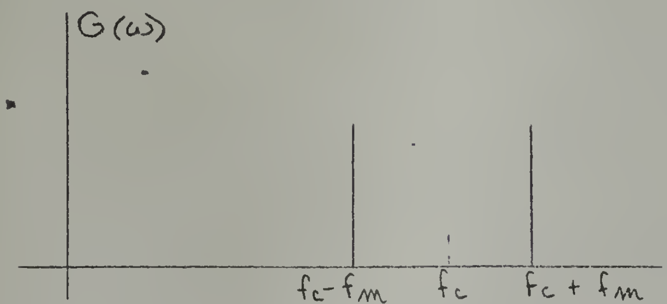
The AM power spectral density for a single modulating tone is:




Only positive frequencies are shown

From the power spectral density it can be seen that even with m = 1, half of the total power of the signal is in the carrier, which does not carry any useful information. Of the remaining power, half of this power is in a redundant side band and duplicates the information carried in the other side band. From the standpoint of efficient utilization of power, AM is also very susceptible to noise and only works well under high signal-to-noise ratios. Under high signal-to-noise ratios, AM does have the advantage of being simple to produce and receive, though its poor peak value rating makes the transponder design difficult. The bandwidth of the AM signal is two times the highest frequency note and in our case equals $2f_m$

A modified form of straight AM is Amplitude Modulation-Double Side Band (AM-DSB). In effect, the carrier is suppressed and all the power is carried by the sidebands. The power spectral density is shown below for a single modulating tone:

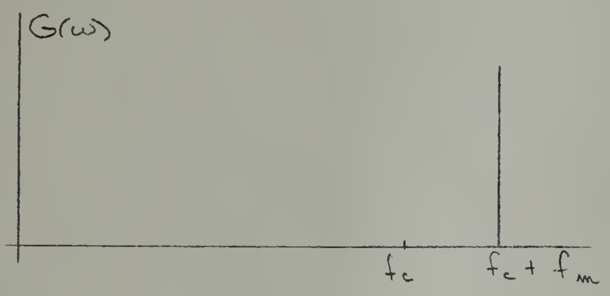



Only positive frequencies are shown

Again, one side band is redundant in that all the information is carried by one side band. This redundancy can be used to advantage through the use of correlation, which will give a potential gain of 3db to be used to combat ambient noise. The bandwidth of an AM-DSB signal is $2f_m$ wide, similar to the AM case.

AM-DSB has important advantages over the AM case, and its only obvious disadvantage is that there is an increase of complexity in the receiving equipment.

A modified form of AM-DSB is Amplitude Modulated - Single Side Band (AM-SSB). In effect only one of the sidebands is transmitted; and therefore, all the useful power is transmitted in a nonredundant sideband. The power spectral density would look as follows:




Only the positive frequency is shown

The bandwidth of the AM-SSB is half of the AM-DSB case and is equal to the highest frequency tone modulating the carrier. In comparing the signal-to-noise ratios of AM-DSB versus AM-SSB, it is found that they are equal. The bandwidth of the AM-SSB is one-half of AM-DSB. AM-SSB has the advantage that it can avoid a problem inherent in AM-DSB; that is the unsymmetrical phase shift of either of the sideband components which would prevent correlation, and therefore, reduce its effectiveness against random noise.

From the above statements, it can be seen that AM-SSB has all the advantages of AM or AM-DSB and lacks many of the disadvantages inherent in the other two forms. In considering linear modulation only AM-SSB will be considered in deciding on the final form of the signal to be sent to the space vehicle.

The two forms of exponential modulation which will be considered in the evaluation process are Frequency Modulation (FM) and Phase Modulation (PM). These two forms are very similar to each other, and they both have the same basic advantages and disadvantages. The useful property of FM or PM is that if an increase in bandwidth can be allowed, then the signal-to-noise ratio can be improved. Since the information of a FM or PM signal is carried in the phase characteristics of the signal, it can be amplitude limited. The limiting of the signal will, among other advantages, help reduce the peak-factor and hence result in a very efficient transponder. Class "C" amplification can be used for an FM or PM signal both on the ground and within the transponder, which greatly improves power utilization. An FM or PM signal is much easier to generate than a corresponding AM-SSB signal.

The two forms of modulation which will be studied to greater detail in section 4 (using the six tone harmonic signals discussed previously) will be AM-SSB and PM.

## 4. Modulation Analysis

As a result of the process of elimination conducted in section three, two forms of modulation (AM-SSB and PM) will be studied in greater detail to help decide on the most favorable method to be used in the tracking of space vehicles. The analysis is done in three steps as follows:

(1) A brief review of the noise power spectrum at the input to the receiver is made.

(2) AM-SSB is analyzed.

(3) PM is analyzed.

The following assumptions and expressions are used throughout the analysis:

(1) Six harmonic tones are chosen for the basic event--32 cps, 160 cps, 800 cps, 4 KC, 20 KC, and 100 KC.

(2) The carrier is 2 KMC.

(3) The final signals, independent of the form of their modulation, will have the same total transmitted power.

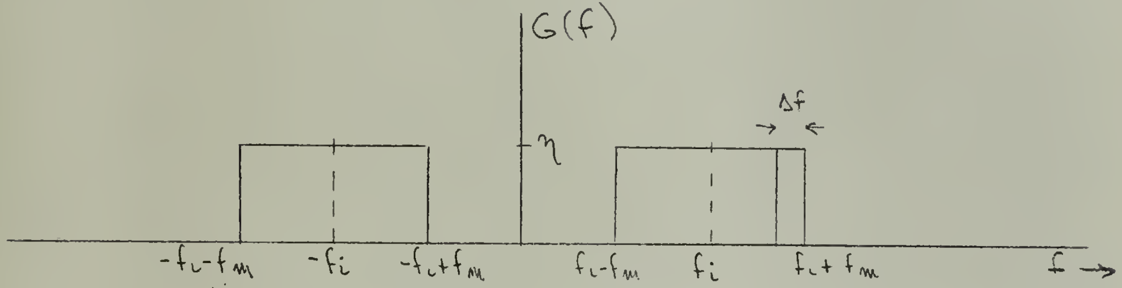
(4) When the power spectrum is drawn usually only positive frequencies will be considered.

(5) Signal-to-noise ratio (S/N) will always be the ratio of the average signal power to the average noise power. All power measurements are on a one-ohm basis. The average signal power is measured in the absence of noise, and the mean noise power is in the presence of an unmodulated carrier.

A. Noise Power Spectrum.[8]

The most important noise that must be contended with is Thermal noise. Thermal noise has a gaussian distribution in time and has a uniform spectrum over the i-f bandwidth of the receiver.

The power spectral density for i-f band limited white noise is shown below:




$$G(\omega) \triangleq \lim_{T \to \infty} \frac{1}{T} \left| X_T(\omega) \right|^2$$

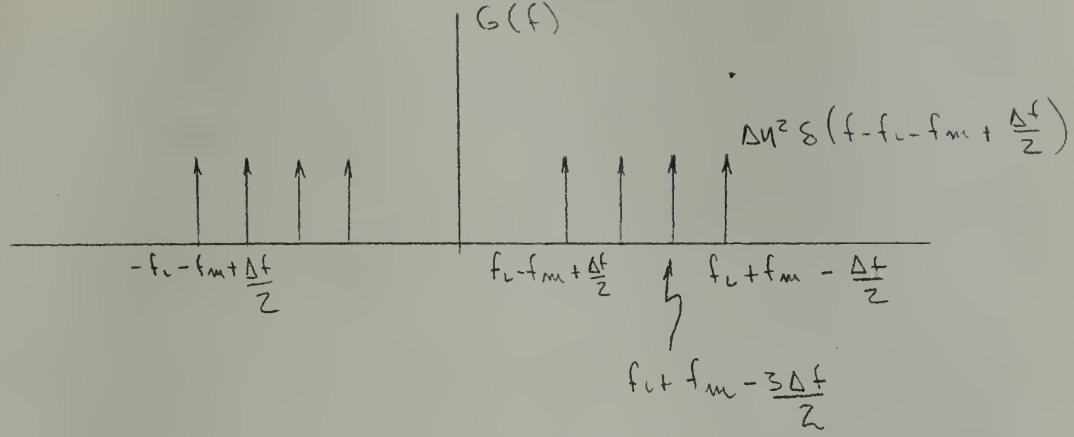
= The "power spectral density"

T = period

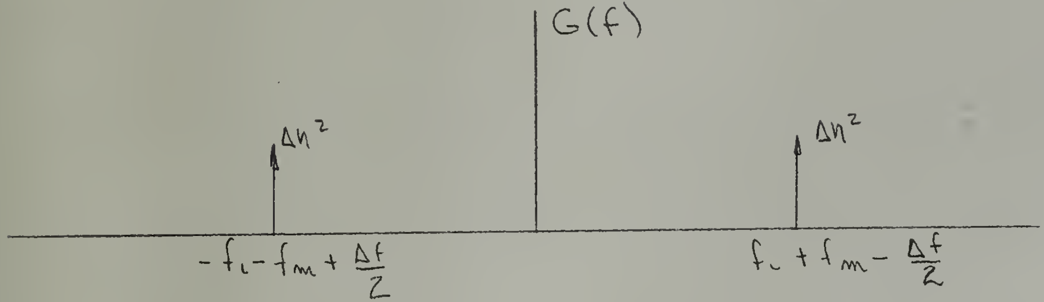
$$X_T(\omega) = \int_{-\infty}^{\infty} x(t)\, e^{-j\omega t}\, dt$$

For our analysis it is convenient to replace a strip of noise $\Delta f$ wide with a discrete noise term at a single frequency with voltage amplitude $\Delta n$. This discrete noise spectrum is shown below:

[8] J. C. Hancock, op. cit., pp. 41-42.



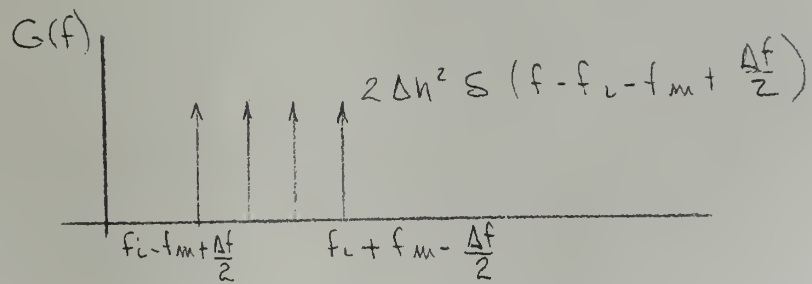

There are $\frac{2f_m}{\Delta f}$ such discrete components, and they occur in pairs. An example of one discrete component is shown below:




This spectrum is the transform of

$$n(t) = 2\,\Delta n \cos 2\pi \left(f_c + f_m - \frac{\Delta f}{2}\right) t$$

If it is restricted to positive frequencies only, the power spectral density would be as such:



The expression for the sum of all the discrete noise components is:

$$n(t) = \sum_{k=1}^{2f_m/\Delta f} 2\Delta n \cos 2\pi \left( f_i + f_m - \frac{(2k-1)\Delta f}{2} \right) t$$

The mean square value of n(t) -- the discrete approximation is:

$$\text{mean square} = \sum_{k=1}^{2f_m/\Delta f} 2(\Delta n)^2 = \frac{4\Delta n^2 f_m}{\Delta f}$$

The mean square value of the actual noise is $4 f_m \eta$.

Equating the two mean square values

$$\frac{4\Delta n^2 f_m}{\Delta f} = 4 f_m \eta$$

$$\therefore \quad \Delta n^2 = \eta \Delta f$$

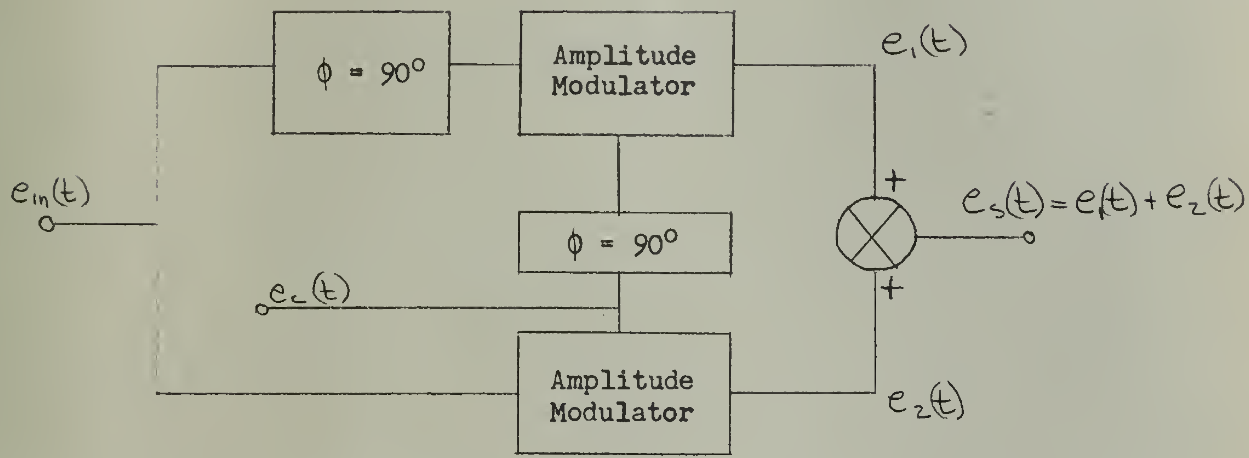
This discrete expression for noise will aid in arriving at the noise output in the analysis of AM-SSB and PM.

B. <u>AM-SSB Analysis</u>.

Two variations of AM-SSB will be analyzed. First all 6 tones will directly modulate a 2 KMC carrier, and secondly, the lowest 5 tones will modulate the 100 KC tone. Then this signal with the 100 KC reinserted will modulate the 2 KMC.

To simplify the analysis of the complex signals mentioned above, a review of the So/No for AM-SSB will be made for a single modulating tone. Then this basic principle is applied to the complex signal.

A simplified method of producing the AM-SSB signal is to use a phase shift method as shown below:[9]




Letting: $$e_{in}(t) = E_m \cos \omega_m t$$

$$e_c(t) = \cos \omega_c t$$

then:

$$e_1(t) = E_m \sin \omega_m t \sin \omega_c t$$

$$e_2(t) = E_m \cos \omega_m t \cos \omega_c t$$

$$e_s(t) = e_1(t) + e_2(t)$$

$$\therefore e_s(t) = E_m \sin \omega_m t \sin \omega_c t + E_m \cos \omega_m t \cos \omega_c t$$

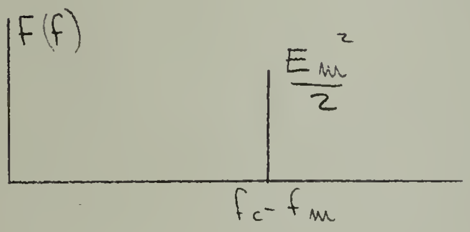
but: $$\sin x \sin y = \cos(x - y) - \cos x \cos y$$

[9] Ibid., p. 48.

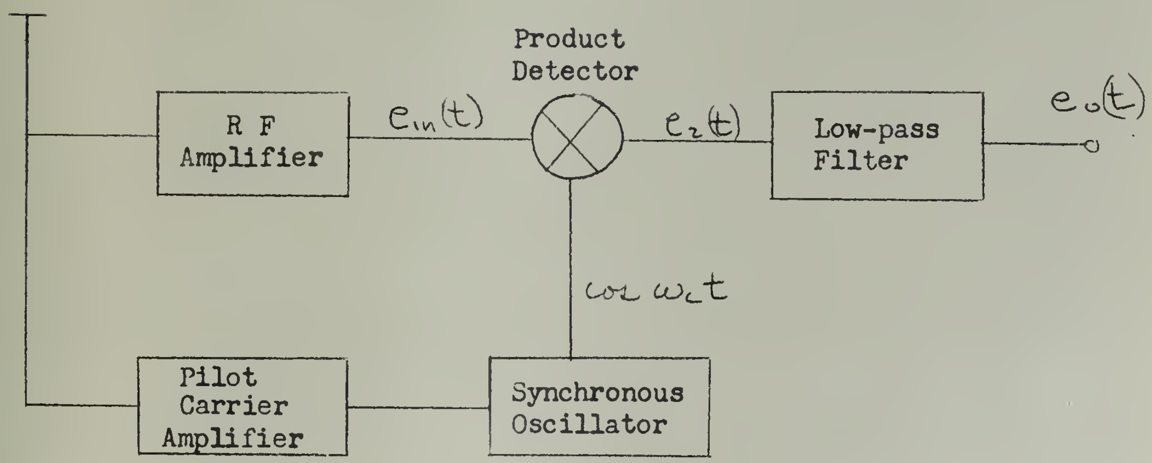
Therefore, substituting we have

$$e_s(t) = E_m \cos(\omega_c - \omega_m)t - E_m \cos\omega_c t \cos\omega_m t$$
$$+ E_m \cos\omega_c t \cos\omega_m t$$

The Power Spectrum of $e_s(t)$ is:




Coherent detection, and in this case synchronous detection is used to detect AM-SSB signals. A pilot carrier of 10% is transmitted, so that $\cos\omega_c t$ at the receiver can be accurately known. A simplified version of the detector is as follows:




$$e_{in}(t) = E_m \cos(\omega_c - \omega_m)t$$
$$e_2(t) = E_m \cos(\omega_c - \omega_m)t \cos\omega_c t$$

but: $\cos x \cos y = \frac{1}{2}\cos(x+y) + \frac{1}{2}\cos(x-y)$

$$\therefore \; e_2(t) = \frac{E_m}{2}\cos(\omega_c - \omega_m + \omega_c)t + \frac{E_m}{2}\cos(\omega_c - \omega_m - \omega_c)t$$

$$= \frac{E_m}{2}\cos(2\omega_c - \omega_m)t + \frac{E_m}{2}\cos\omega_m t$$

after the low pass filter:

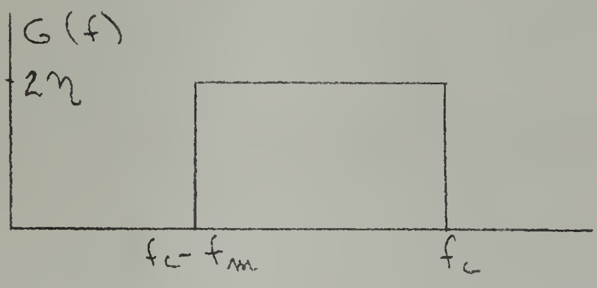
$$e_2(t) = \frac{E_m}{2}\cos\omega_m(t)$$

The total signal in to the detector is:

$$e_{in}(t) = E_m \cos(\omega_c - \omega_m)t + n(t)$$

Therefore, investigation of the form of n(t), and its available strength before and after detection is made.

The actual noise spectrum at the input to the AM-SSB receiver is as follows:




The noise signal at the input to the detector was shown to be represented by:

$$n(t) = \sum_{k=1}^{f_m/\Delta f} 2\Delta n \cos 2\pi\left(f_c - \frac{(2K-1)\Delta f}{2}\right)t$$

Therefore, at the product detector for the noise signal we have:

$$n_2(t) = \sum_{k=1}^{f_m/\Delta f} 2\,\Delta n \cos 2\pi\left(f_c - \frac{(2K-1)\Delta f}{2}\right)t \cos\omega_c t$$

$$n_2(t) = \sum_{k=1}^{f_m/\Delta f} \Delta n \cos 2\pi\left(2f_c - \frac{(2K-1)}{2}\Delta f\right)t$$

$$+ \Delta n \cos 2\pi\left[\frac{(2K-1)\Delta f}{2}\right]t$$

After the low-pass filter

$$n_o(t) = \sum_{k=1}^{f_m/\Delta f} \Delta n \cos 2\pi\left\{\frac{(2K-1)\Delta f}{2}\right\}t$$

Then the total signal out is:

$$e_o(t) = \frac{E_m}{2}\cos\omega_m t + \sum_{k=1}^{f_m/\Delta f} \Delta n \cos 2\pi\left\{\frac{(2K-1)\Delta f}{2}\right\}t$$

$$S_o = \frac{1}{T}\int_{-T/2}^{T/2}\left(\frac{E_m}{2}\cos\omega_m t\right)^2 dt = \frac{E_m^{\,2}}{8}$$

Likewise, each discrete term has a mean square value of $\frac{\Delta n^2}{2}$.
and there are $f_m/\Delta f$ of them.

$$\therefore N_o = \frac{\Delta n^2}{2} \frac{f_m}{\Delta f} = \frac{\eta \Delta f f_m}{2 \Delta f}$$

since

$$\Delta n^2 = \eta . \Delta f$$

$$\therefore \frac{S_o}{N_o} = \frac{E_m^2/8}{\eta f_m/2} = \frac{E_m^2}{4 \eta f_m}$$

The value of

$$S_{IN} = \frac{E_m^2}{2}$$

The value of

$$N_{IN} = 2 \eta f_m$$

and $$\frac{S_{IN}}{N_{IN}} = \frac{E_m^2}{4 \eta f_m}$$

$$\therefore \frac{S_{IN}}{N_{IN}} = \frac{S_o}{N_o}$$

Therefore, the pre-detection and post-detection signal-to-noise ratios are equal.

Since the above is true, the next two systems will be compared based on the pre-detection signal-to-noise ratios.

Analysis of So/No with all six tones modulating the 2 KMC carrier will now be considered.

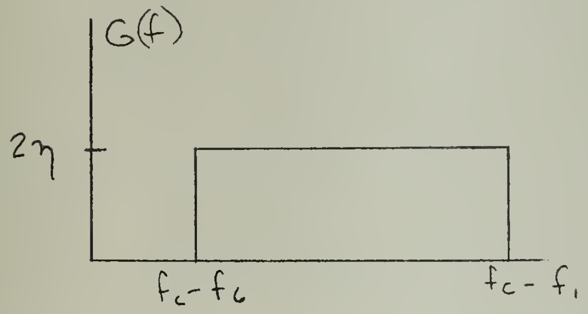
| | | | | | |
|---|---|---|---|---|---|
| Let $f_1$ | = | 32 cps | $f_4$ | = | 4 KC |
| $f_2$ | = | 160 cps | $f_5$ | = | 20 KC |
| $f_3$ | = | 800 cps | $f_6$ | = | 100 KC |
| and $f_c$ | = | 2 KMC | | | |

The signal at the input of the receiver is:

$$e_s(t) = E_m \sum_{j=1}^{6} \cos(\omega_c - \omega_j)t + \frac{E_m}{10} \cos \omega_c t$$

The power spectrum of $e_s(t)$ for all six tones modulating the 2 KMC carrier is shown in illustration #1.

The spectrum of $n_{in}(t)$ is:




$P_t$ = power transmitted = $6 \times \frac{E_m^2}{2} + \frac{E_m^2}{200}$

Effective $\quad S_{IN} = 6 \times \frac{E_m^2}{2} = 3E_m^2$

$$N_{in} = 2\eta(f_c - f_1 - f_c + f_6) = 2\eta(f_6 - f_1)$$

$$= 2\eta(100{,}000 - 32) \approx 2\eta\, 100\,KC$$

$$\frac{S_o}{N_o} = \frac{S_{IN}}{N_{IN}} = \frac{3E_m^2}{2\eta\, 100\,KC}$$

The second analysis is for 5 tones amplitude modulating (single side band) the 100 KC tone, and then reinserting the 100 KC tone. This set of tones amplitude modulates (single side band) the 2 KMC carrier. The results of the first submodulation with the 100 KC tone reinserted, can be written as:

$$e_{s1}(t) = E_m \sum_{j=0}^{5} \cos(\omega_6 - \omega_j)$$

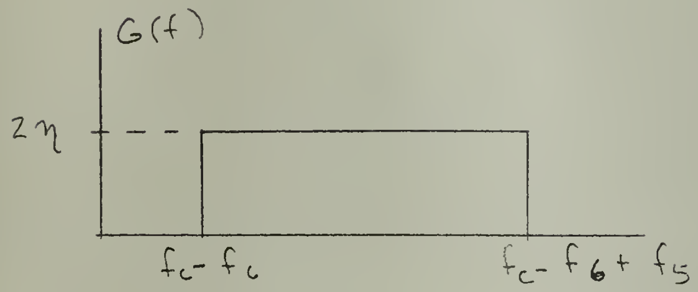
The power spectral density of $e_{s1}(t)$ is shown in illustration #2.

AM-SSB the 2 KMC carrier with $e_{s1}(t)$ and reinserting a 10% carrier component will give:

$$e_{s2}(t) = E_m \sum_{j=0}^{5} \cos(\omega_c - \omega_6 + \omega_j) + \frac{E_m}{10}\cos\omega_c t$$

The power spectral density of $e_{s2}(t)$ is shown in illustration #3.

The effective power spectrum of n in(t) is:




$$P(t) = 3E_m^2 + .005\, E_m^2$$

$$\text{Effective} \quad S_{IN} = 3E_m^2$$

$$N_{IN} = 2\eta(f_c - f_6 + f_5 - f_c + f_c) = 2\eta f_5$$

$$= 2\eta\ 20\,KC$$

$$\frac{S_o}{N_o} = \frac{S_{IN}}{N_{IN}} = \frac{3E_m^2}{2\eta(20\,KC)}$$

$$\frac{\frac{S_o}{N_o}\ (\text{for } 2^{nd} \text{ case})}{\frac{S_o}{N_o}\ (\text{for } 1^{st} \text{ case})} = \frac{100\,KC}{20\,KC} = 5$$

$$= 6.98 \text{ db}$$

improvement for the second form of modulation.

C. PM Analysis.

In the analysis of Phase Modulation, certain parameters will be varied, and for each variation a signal-to-noise ratio and spectrum analysis will be conducted. The different PM studies to be made are:

(1) PM with the six tones of equal amplitude, and the phase deviations index equal to 0.7.

(2) PM with the six tones of equal amplitude and the phase deviation index equal to 1.0.

(3) PM with the six tones of equal amplitude and the phase deviation index equal to 1.5.

(4) PM with the six tones being weighted and the phase deviation for the 100 KC tone equal to .99.

(5) PM with the six tones being weighted and the phase deviation for the 100 KC tone equal to 2.18.

A quick review of phase modulation and demodulation, and the detection of noise will be covered before the five different combinations mentioned above are analyzed. Phase modulation is defined as a form of angle modulation in which the linearly increasing phase of $\omega_c t + \Phi_c$ has added to it a time varying phase angle that is proportional to the applied modulating wave.[10]

[10] H. S. Black, Modulation Theory, D. Van Nostrand Co. Inc., Princeton, New Jersey, p. 183, 1953.

If $f_m(t)$ = modulating wave = $\Phi_v \cos \omega_v t = E_m \cos \omega_v t$

$\Phi_V$ = phase deviation index

$f_c(t)$ = unmodulated carrier = $E_c \cos \theta t$

$$= E_c \cos(\omega_c t + \Phi_c)$$

$$\theta(t) = \omega_c t + \Phi_c + f_m(t) \qquad \text{for PM}$$

Letting $\Phi_c = 0$

Then $f_s(t)$ = modulated carrier = $E_c \cos[\omega_c t + \Phi_v \cos \omega_v t]$

but $\cos(A+B) = \cos A \cos B - \sin A \sin B$

$$\therefore \quad f_s(t) = E_c[\cos \omega_c t \cos(\Phi_v \cos \omega_v t) - \sin \omega_c t \sin(\Phi_v \cos \omega_v t)]$$

$$\cos(x \sin y) \equiv J_0(x) + 2 \sum_1^\infty J_{2m}(x) \cos 2my$$

$$\sin(x \sin y) \equiv 2 \sum_1^\infty J_{2m-1}(x) \sin(2m-1) y$$

$$\therefore \quad f_s(t) = E_c\left[\cos \omega_c t \left(J_0[\Phi_v] + 2 \sum_1^\infty J_{2m}(\Phi_v) \cos 2m\omega_v t\right) - 2 \sin \omega_c t \left(\sum_1^\infty J_{2m-1}(\Phi_v) \sin(2m-1)\omega_v t\right)\right]$$

$$= E_c \{(\cos \omega_c t) J_0(\Phi_v) + 2 J_2(\Phi_v) \cos \omega_c t \cos 2\omega_v t - 2 J_1(\Phi_v) \sin \omega_c t \sin \omega_v t + 2 J_4(\Phi_v) \cos \omega_c t \cos 4\omega_v t - 2 J_3(\Phi_v) \sin \omega_c t \sin 3\omega_v t \ldots\}$$

$$2 \cos X \cos y = \cos (X + y) + \cos (X - y)$$

$$2 \sin X \sin y = \cos (x - y) - \cos (x + y)$$

$$f_s(t) = J_0(\phi_v) E_c \cos \omega_c t + J_1(\phi_v) \cos (\omega_c + \omega_v) t \, (E_c)$$

$$- J_1(\phi_v) E_c \cos (\omega_c - \omega_v) t + J_2(\phi_v) E_c \cos (\omega_c + 2\omega_v) t$$

$$+ J_3(\phi_v) E_c \cos (\omega_c + 3\omega_v) t - J_3(\phi_v) E_c \cos (\omega_c - 3\omega_v) t$$

$$- \; . \; . \; . \; . \; . \; . \; . \; .$$

For more than one sinusoidal modulating term, the expression for $f_s(t)$ can be derived as shown below:[11]

$$f_m(t) = \sum_{v=1}^{v} \phi_v \sin \omega_v t$$

then for a phase modulated wave

$$f_s(t) = E_c \sin \left[ \omega_c t + \sum_{v=1}^{v} \phi_v \sin \omega_v t \right]$$

Imaginary component $\equiv$ Im

$$f_s(t) = \operatorname{Im} E_c e^{j(\omega_c t + \sum_{v=1}^{v} \phi_v \sin \omega_v t)}$$

$$= \operatorname{Im} E_c e^{j\omega_c t} e^{j \sum_{v=1}^{v} \phi_v \sin \omega_v t}$$

[11] Ibid., p. 195.

but $$e^{jx\sin\theta} \equiv \sum_{-\infty}^{\infty} J_n(x)\, e^{jn\theta}$$

$$f_s(t) = \mathrm{Im}\left\{E_c e^{j\omega_c t} \sum_{-\infty}^{\infty} J_1(\phi_1) e^{-j\omega_1 t} \sum_{-\infty}^{\infty} J_2(\phi_2) e^{j2\omega_2 t} \cdots\right\}$$

$$= \mathrm{Im}\left\{E_c e^{j\omega_c t} \prod_{n=1}^{n} \sum_{n_v=-\infty}^{\infty} J_{n_v}(\phi_{n_v}) e^{jn_v\omega_{n_v}t}\right\}$$

$$= \mathrm{Im}\left\{E_c e^{j\omega_c t} \sum_{n_v=-\infty}^{\infty} \prod_{v=1}^{v} J_{n_v}(\phi_{n_v}) \prod_{v=1}^{v} e^{jn_v\omega_{n_v}t}\right\}$$

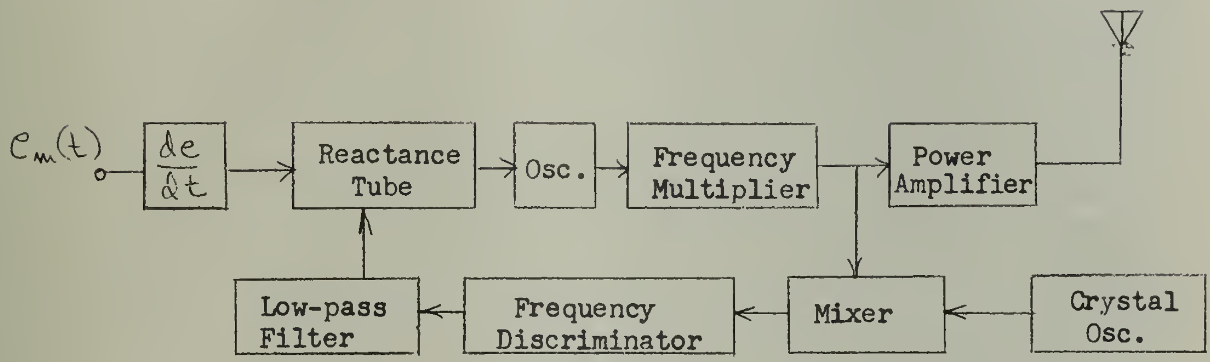
$$= \mathrm{Im}\left\{E_c \sum_{n_v=-\infty}^{\infty} \prod_{v=1}^{v} J_{n_v}(\phi_{n_v}) e^{j\omega_c t} e^{j\sum_{v=1}^{v} n_v\omega_v t}\right\}$$

$$= E_c \sum_{n_v=-\infty}^{\infty} \left[\prod_{v=1}^{v} J_{n_v}(\phi_v)\right] \sin\left(\omega_c t + \sum_{v=1}^{v} n_v\omega_v t\right)$$

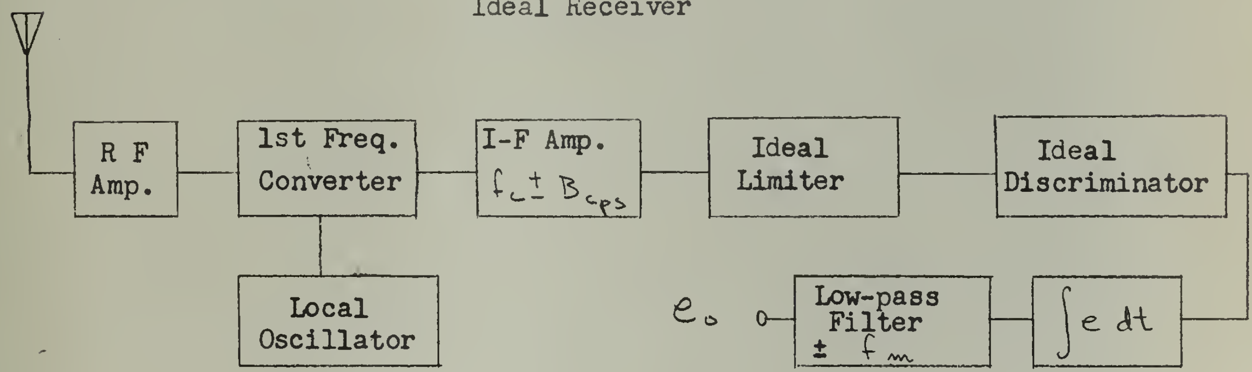
$$J_{-n}(x) \equiv (-1)^n J_n(x)$$

A basic method of producing and receiving a PM wave is shown below:[12]

Transmitter

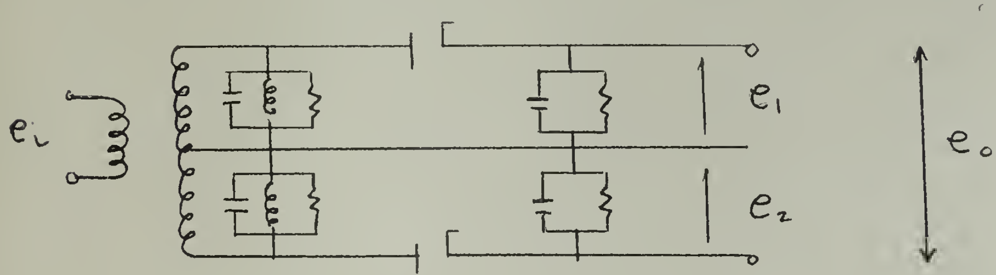



Ideal Receiver




$f_m < B$

A more detailed look at one form of a discriminator is shown below:




[12]J. C. Hancock, op. cit., pp. 52-53.

The next step is to find an expression for the So/No of an ideal PM receiver. The following assumptions will apply throughout the signal-to-noise analysis.

(1) The signal-to-noise ratio at the input of the receiver must be large. Evidence has shown that if the S/N ratio is less than 13 db that breaking through the limiter begins to be noticed. A minimum of 15 db carrier-to-noise ratio will be used. $B_{15\,DB} \equiv$ maximum i-f bandwidth which will meet 15 db requirement.

(2) The bandwidth of the i-f amplifier is $f_c = \pm B$ cps.

(3) The discriminator gives an output directly proportional to the instantaneous frequency of the signal.

(4) After the signal goes through the integrator, it goes through a low pass filter whose bandwidth (fm) is equal to $\pm$ 20 KC (80 KC —— 100 KC). The bandwidth of the low pass filter is less than the bandwidth of the i-f amplifier.

(5) If the modulating signal is $f_m(t) =$

$$E_m \cos \omega_m t = \phi_v \cos \omega_m t$$

then the phase modulated carrier measured at the output of the i-f amplifier is $f_s(t) = E_c \cos\left[\omega_c t + E_m \cos \omega_m t\right]$

The instantaneous frequency is given by

$$\omega = \frac{d\theta}{dt} = \omega_c - E_m \omega_m \sin \omega_m t$$

Since the discriminator output is proportional to the instantaneous frequency deviation away from $\omega_c$, or $\omega - \omega_c$, the frequency

deviation (fd) is:

$$f_d(t) = -bE_m\,\omega_m \sin \omega_m t$$

b = constant of discriminator

At the output of the integrator

$$f_o(t) = \int^t f_d(t)\,dt = bE_m \cos \omega_m t$$

Therefore, the final signal out is equal to the modulating signal.

(6) The PM modulation index $\Phi_v$ is proportional to the frequency deviation which in turn is proportional to the i-f bandwidth = B.[13]

$$\Phi_v = 2B$$

(7) The effective i-f bandwidth for distortionless transmission is defined as that bandwidth ($B_{.99}$) which will pass 99.0% of the total signal power.

(8) The phase deviation index effective ($\Phi_{veff}$) is defined as:

$$\Phi_{veff} = \frac{\Phi_v}{B_{.99}}\,B_{15DB} \qquad (\Phi_{veff} \leq \Phi_v)$$

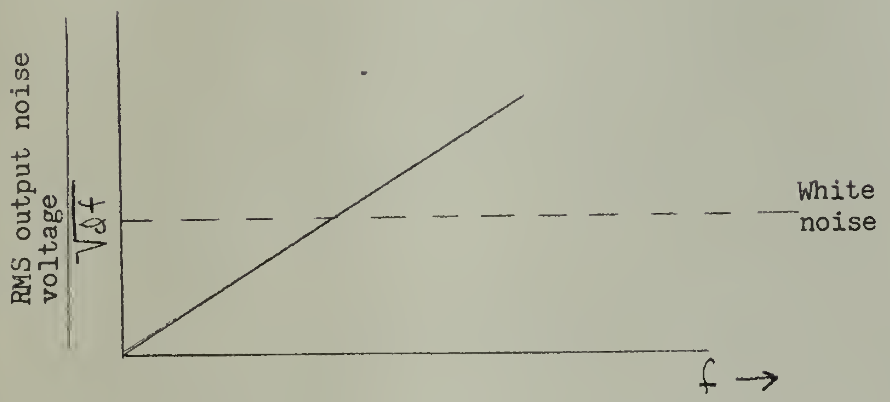
(9) The average output signal power is:

$$S_o = \frac{b^2\,(\Phi_{v\,eff})^2}{2} \quad \text{watts}$$

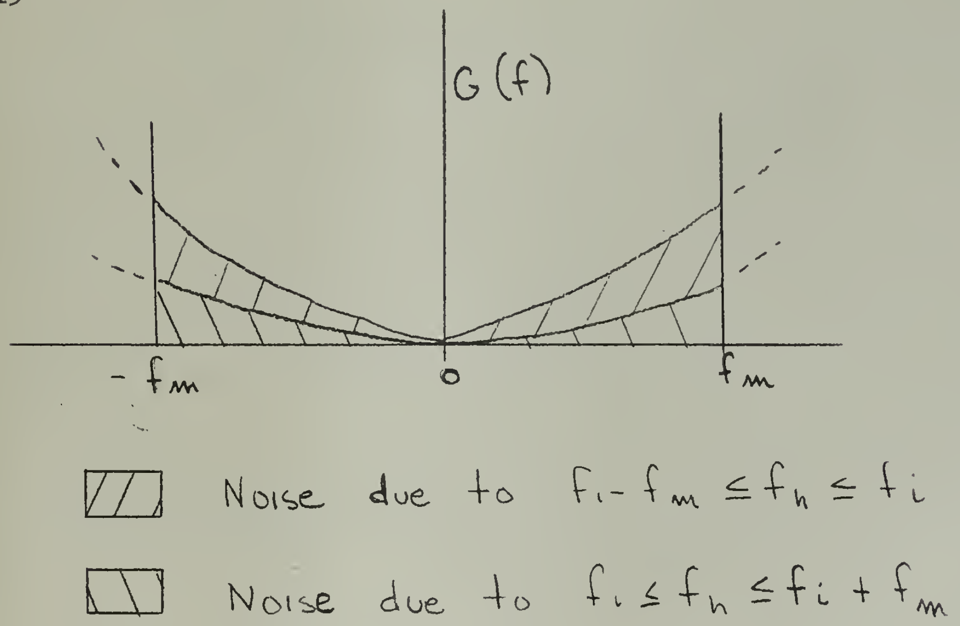
Since amplitude variations have been reduced by the limiter, a phase modulator detector is only sensitive to the phase of the noise signal.

[13] M. Schwartz, op. cit., p. 315.

The spectrum of rms noise voltage at the output of a PM receiver is:[14]




The power spectrum of the noise at the output of the PM receiver is:[15]




[14] M. Schwartz, op. cit., p. 302.

[15] J. C. Hancock, op. cit., p. 58.

Under the assumption of large carrier-to-noise level, PM noise acts as narrow-band-noise and can be handled as a linear system and superposition applies.[16]

The total noise at the output of the low pass filter is given by:[17]

$$N_o = \int_{-f_m}^{f_m} dN_o = \eta \left(\frac{b}{E_c}\right)^2 \int_{-f_m}^{f_m} \omega^2 \, df$$

$$= \frac{2\eta}{3} \cdot \left(\frac{b}{E_c}\right)^2 (\omega_m)^2 f_m \qquad (\omega_m = 2\pi f_m)$$

It can be seen that the noise out is effectively controlled by the bandwidth of the low pass filter and is independent of the i-f bandwidth, as long as there is a large carrier-to-noise level.

The final So/No is therefore given by:[18]

$$\left(\frac{S_o}{N_o}\right)_{PM} = 3 \, \Phi_{v\,eff}^2 \, \frac{S_{c\ i\text{-}f}}{2\eta f_m}$$

$S_c$ i-f = Average power passed by the i-f bandwidth (± B)

[16] M. Schwartz, op. cit., p. 302

[17] Ibid., p. 302.

[18] Ibid., p. 303.

With this basic background on PM, the analysis of the five specific combinations mentioned above will be made. Similar to the second variation in the AM-SSB case, the first five tones amplitude modulate (single side band) the 100 KC tone, and then the 100 KC tone is reinserted. Then these six tones phase modulate the carrier. The following notation is used:

$f_c$ = 2 KMC

$f_6$ = 100 KC

$f_5$ = 100 KC - 32 cps = 99,968 cps

$f_4$ = 100 KC - 160 cps = 99,840 cps

$f_3$ = 100 KC - 800 cps = 99,200 cps

$f_2$ = 100 KC - 4 KC = 96 KC

$f_1$ = 100 KC - 20 KC = 80 KC

Then the phase modulated wave is:

$$f_s(t) = Ec \sin\left[\omega_c t + \sum_{v=1}^{6} \phi_v \sin \omega_v t\right]$$

$$f_s(t) = Ec \sum_{n_v=-\infty}^{\infty} \left[\prod_{v=1}^{v} J_{n_v}(\phi_v)\right] \sin\left(\omega_c t + \sum_{r=1}^{6} n_v \omega_v t\right)$$

$$= E_c \sum_{i=-\infty}^{\infty} \sum_{j=-\infty}^{\infty} \sum_{k=-\infty}^{\infty} \sum_{l=-\infty}^{\infty} \sum_{m=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} J_i(\phi_i) J_j(\phi_j) J_k(\phi_k) J_l(\phi_l) J_m(\phi_m) J_n(\phi_n) \sin(\omega_c t + i\omega_1 t + j\omega_2 t + k\omega_3 t + l\omega_4 t + m\omega_5 t + n\omega_6 t)$$

To obtain the magnitude of the voltage and the power for each frequency term in three of the spectrum analyses, where the tones had equal weight, a Fortrain program was used to program the 1604 computer. This Fortran program is attached as appendix I and is called Program #1.

To keep the computer from running too long the signs of the voltage terms are all positive.

In the usual spectrum analysis of PM or FM waves where only one tone is modulating the carrier, all voltage terms whose magnitudes are less than .01 are discarded. It is felt that this requirement is too arbitrary, and that it is possible that a smaller voltage magnitude should be considered. It is of interest to find just how small in magnitude a voltage term should be before its contributions to the signal power were negligible. A Fortran program was used to obtain this information. Again $X(1) \longrightarrow X(7)$ is for $\phi_v = 1.5$ and these would be changed for different phase deviation indexes. This Fortran program is attached as appendix II and is called Program #2.

To enable making So/No calculations for the PM case, it was necessary to know how the power was distributed with respect to frequency. A Fortran program is used to obtain this information. This Fortran program is attached as appendix III and is called Program #3.

The first case to be analyzed is for phase modulation of the carrier with the six tones of equal amplitude, and the phase deviation index = .7

$$f_s(t) = E_c \sum_{i=-\infty}^{\infty} \sum_{j=-\infty}^{\infty} \sum_{k=-\infty}^{\infty} \sum_{l=-\infty}^{\infty} \sum_{m=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} J_i(.7)\, J_j(.7)\, J_k(.7)\, J_l(.7)\, J_m(.7)\, J_n(.7) \sin(\omega_c + i\omega_1 + j\omega_2 + k\omega_3 + l\omega_4 + m\omega_5 + n(\omega_6)\, t$$

$J_0(.7) = .8812$ $\qquad$ $J_1(.7) = .3290$

$J_2(.7) = .0588$ $\qquad$ $J_3(.7) = .0069$

The results of Program #1 are attached in a separate envelope. The results of Program #2 showing the amount of power included in the spectrum as the magnitude of the voltage term is decreased are shown below. The carrier voltage is normalized to one.

The average power of the carrier $= \frac{Ec^2}{2} = \frac{1^2}{2} = .5$ watts

Power Contribution as a Function of the Magnitude of the Voltage Term

| Magnitude of Voltage Terms | Terms | Power | % of Total Power |
|---|---|---|---|
| $\lvert V\rvert > .01$ | 365 | .482300 | 96.4% |
| $\lvert V\rvert > .008$ | 605 | .492232 | 98.5% |
| $\lvert V\rvert > .006$ | 605 | .492232 | 98.5% |
| $\lvert V\rvert > .004$ | 1085 | .496782 | 99.4% |
| $\lvert V\rvert > .002$ | 1349 | .498101 | 99.6% |
| $\lvert V\rvert > .001$ | 2296 | .499501 | 99.99% |

In this case, the spectrum is well analyzed by considering only the terms whose voltage magnitudes are greater than, or equal to .01. The spectrum of this modulation is shown in illustration #4. Only the positive frequency terms greater in magnitude than the carrier have been shown to conserve space.

The results of Program #3 are listed below.

Power Contribution as a Function of Bandwidth

| Bandwidth | Power | % of Total Power |
|---|---|---|
| $\pm$ 100,000 cps | .3885 | 77.5% |
| $\pm$ 150,000 cps | .4030 | 80.6% |
| $\pm$ 200,000 cps | .4781 | 95.8% |
| $\pm$ 250,000 cps | .4802 | 96.0% |
| $\pm$ 300,000 cps | .4969 | 99.3% |
| $\pm$ 400,000 cps | .4997 | 99.98% |
| $\pm$ 500,000 cps | .4999 | 99.99% |
| $\pm$ 600,000 cps | .5 | 100% |

A plot of the cumulative power as a function of bandwidth is shown in illustration #5.

In comparing any tracking system, the available transmitter power should be fixed. It is arbitrarily chosen that the carrier-to-noise ratio at the input to the i-f amplifier is 15 db for an i-f bandwidth of

$\pm$ B = 400,000 cycles and $\phi_v = 0.7$

This arbitrary definition sets the magnitude of $\eta$ which is used in all of the following analyses:

S/N ratio (db) = 10 log S/N = 10 log $S/2\eta B$

S for B = $\pm$ 400,000 = .4997 watts

$$15 \text{ db} = 10 \log \frac{.4997}{N}$$

$$\log \frac{.495}{N} = 1.5$$

$$\frac{.495}{N} = 31.6$$

$$N = \frac{.4997}{31.6} = .0158$$

$$\eta = \frac{N}{2B} = \frac{1.58 \times 10^{-2}}{2(.8 \times 10^{6})} = .9875 \times 10^{-8}$$

---

$B_{.99} = \pm$ 270,000 cycles (see illustration #6)

$B_{15db} = \pm$ 400,000 cycles (see above)

$$\phi_{veff} = \frac{\phi_v}{B_{.99}} B_{15db} = \frac{.7\ (\pm 400,000)}{\pm 270,000} \doteq .7$$

since $(\phi_{veff} \leq \phi_v)$

$$\frac{S_c}{2\eta f_m} = \frac{.495}{2(.9875 \times 10^{-8})\,20,000} = 1255$$

$$S_o/N_o = 3(\phi_{veff})^2 \frac{S_c}{2\eta f_m} = 3\,(.7)^2\,1255 = 1845$$

The second case to be analyzed is for PM of the carrier with the six tones of equal amplitude and the phase deviation index = 1.0

$$f_s(t) = E_c \sum_{i=-\infty}^{\infty} \sum_{j=-\infty}^{\infty} \sum_{k=-\infty}^{\infty} \sum_{l=-\infty}^{\infty} \sum_{m=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} J_i(1) J_j(1)$$

$$J_k(1) J_l(1) J_m(1) J_n(1) \sin(\omega_c + i\omega_1$$

$$+ j\omega_2 + k\omega_3 + l\omega_4 + m\omega_5 + n\omega_6)t$$

$J_o(1) = .7652$ $\qquad$ $J_1(1) = .4401$

$J_2(1) = .1149$ $\qquad$ $J_3(1) = .0196$

The results of Program #1 are attached in a separate envelope.

The results of Program #2 showing the amount of power included in the spectrum as the magnitude of the voltage term is decreased are listed in the table below.

The total average power = .5

Power Contribution as a Function of the Magnitude of the Voltage Term

| Magnitude of Voltage Terms | Terms | Power | % of Total Power |
|---|---|---|---|
| $\|V\| > .01$ | 797 | .445828 | 89.2% |
| $\|V\| > .008$ | 1277 | .469690 | 94.0% |
| $\|V\| > .006$ | 1341 | .471379 | 94.3% |
| $\|V\| > .004$ | 2373 | .487939 | 97.7% |
| $\|V\| > .002$ | 3933 | .495312 | 99.3% |
| $\|V\| > .001$ | 6239 | .498311 | 99.7% |

The voltage spectrum is well analyzed by looking at the voltage terms greater than or equal to .01. This spectrum is shown in illustration #6. Only the positive frequency terms greater in magnitude than the carrier are shown.

The results of Program #3 are listed below.

Power Contribution as a Function of Bandwidth

| Bandwidth | Power | % of Total Power |
|---|---|---|
| ± 100,000 | .270697 | 54.1% |
| ± 150,000 | .312467 | 62.5% |
| ± 200,000 | .416324 | 83.3% |
| ± 300,000 | .476629 | 95.5% |
| ± 400,000 | .495079 | 99.0% |
| ± 500,000 | .499340 | 99.8% |
| ± 600,000 | .5 | 100% |

A plot of the cumulative power as a function of bandwidth is shown in illustration #7.

$B_{.99} = \pm 400,000$ cycles (see illustration #7)

For the carrier-to-noise ratio at the input of the receiver to be 15 db, the i-f bandwidth must be

$B_{15\,db} = \pm 396,000$ cycles

since at this bandwidth

$$S = .494$$

$$s/N = 10 \log \frac{S}{2\eta B} = 10 \log \left\{ \frac{.494}{2(.9875 \times 10^{-8})(2 \times 396{,}000)} \right\} = 15 \text{ db}$$

$$\Phi_{v\,eff} = \frac{\Phi_v}{B_{.99}} B_{15\,db} = \frac{1}{\pm 400{,}000} \pm 396{,}000 = .99$$

$$\frac{S_c}{2\eta f_m} = \frac{.494}{2(.9875 \times 10^{-8})\, 2 \times 10^4} = 1250$$

$$\frac{S_o}{N_o} = 3(\Phi_{v\,eff})^2 \frac{S_c}{2\eta f_m} = 3(.99)^2(1250) = 3675$$

The third case to be analyzed is for PM of the carrier with the six tones of equal amplitude, and the phase deviation index = 1.5.

$$f_s(t) = E_c \sum_{i=-\infty}^{\infty} \sum_{j=-\infty}^{\infty} \sum_{k=-\infty}^{\infty} \sum_{l=-\infty}^{\infty} \sum_{m=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} J_i(1.5)$$

$$J_j(1.5)\, J_k(1.5)\, J_l(1.5)\, J_m(1.5)\, J_n(1.5) \cdot$$

$$\sin(\omega_c + i\omega_1 + j\omega_2 + k\omega_3 + l\omega_4 + m\omega_5 + n\omega_6)t$$

$J_0(1) = .5118$ $\qquad$ $J_1(1) = .5579$

$J_2(1) = .2321$ $\qquad$ $J_3(1) = .06096$

The results of Program #1 are attached in a separate envelope.

The results of Program #2 showing the amount of power included in the spectrum as the magnitude of the voltage term is decreased are listed in the table below:

The total average power = 0.5

Power Contribution as a Function of the Magnitude of the Voltage Term

| Magnitude of Voltage Terms | Terms | Power | % of Total Power |
|---|---|---|---|
| $\|V\| > .01$ | 3,033 | .38660 | 77.5% |
| $\|V\| > .005$ | 4,605 | .42732 | 85.7% |
| $\|V\| > .004$ | 8,445 | .46711 | 93.6% |
| $\|V\| > .003$ | 9,849 | .47399 | 95.0% |
| $\|V\| > .002$ | 12,701 | .48260 | 96.7% |
| $\|V\| > .001$ | 25,341 | .49553 | 99.3% |
| $\|V\| > .0005$ | 37,221 | .49779 | 99.6% |
| $\|V\| > .0001$ | 67,681 | .4988 | 99.8% |

For the first time evidence shows that considering only those voltage terms greater than .01 in magnitude leaves out 22.5% of the power. Roughly all voltage terms greater than .0044 in magnitude should be considered, and this still only gives 90% of the total power of the PM signal.

Due to the enormous amount of terms involved, only those terms greater than .01 in magnitude are entered in the spectrum density shown in illustration #8.

The results of Program #3 are listed below:

Power Consideration as a Function of Bandwidth

| Bandwidth | Power | % of Total Power |
|---|---|---|
| ± 175,000 | .22819 | 56.4% |
| ± 275,000 | .34260 | 68.5% |
| ± 375,000 | .42056 | 84.0% |
| ± 475,000 | .46571 | 93.2% |
| ± 575,000 | .48681 | 97.3% |
| ± 675,000 | .49483 | 98.8% |
| ± 775,000 | .49688 | 99.3% |
| ± 875,000 | .49728 | 99.5% |

A plot of the cumulative power as a function of bandwidth is shown in illustration #9.

The i-f bandwidth for a carrier-to-noise ratio at the input of the receiver of 15 db is $B_{15db}$ = 275,000 cycles.

since:

S = .3426

$$\frac{S}{N} = 10 \log \frac{S_{c\,\iota\iota}}{2\eta B} = 10 \log \left\{ \frac{.3426}{2(.9875 \times 10^{-8})(2 \times 275{,}000)} \right\} = 15 \text{ db}$$

$$B_{.99} = \pm 695{,}000 \text{ cycles}$$

$$\Phi_{veff} = \Phi_v \frac{B_{15\,db}}{B_{.99}} = \frac{1.5\,(\pm 275{,}000)}{(\pm 695{,}000)} = .595$$

$$\frac{S_c}{2\eta f_m} = \frac{.3426}{2(.9875 \times 10^{-8})(2 \times 10^{4})} = 865$$

$$S_o/N_o = 3\,(\Phi_{veff})^2\, S_c/2\eta f_m = 3(.595)^2\, 865 = 920$$

In the discussion of incremental tracking versus total tracking, it was mentioned that the tones could be weighted in accordance with their use and importance. The following weightings were given to the tones based on this use and importance.

| Frequency | Relative Power | Relative Voltage A | Relative Voltage B |
|---|---|---|---|
| 100 KC | .98 | .99 | 2.18 |
| 99,968 | .012 | .11 | .242 |
| 99,840 | .002 | .045 | .1 |
| 99,200 | .002 | .045 | .1 |
| 96,000 | .002 | .045 | .1 |
| 80,000 | .002 | .045 | .1 |

The fourth case to be analyzed is for PM of the carrier with six tones whose relative amplitudes are shown above and the phase deviation index for the 100 KC term is $\phi_v = .99$

$$f_s(t) = E_c \sum_{i=-\infty}^{\infty} \sum_{j=-\infty}^{\infty} \sum_{k=-\infty}^{\infty} \sum_{l=-\infty}^{\infty} \sum_{m=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} J_i(.045)$$

$$J_j(.045)\, J_k(.045)\, J_l(.045)\, J_m(.11)\, J_n(.99)$$

$$\sin(\omega_c + i\omega_1 + j\omega_2 + k\omega_3 + l\omega_4 + m\omega_5 + n\omega_6)t$$

The Bessel functions values used in this analysis are listed below:

| X | $J_0$ | $J_1$ | $J_2$ | $J_3$ | $J_4$ |
|---|---|---|---|---|---|
| .99 | .7696 | .4368 | .1128 | .01900 | .002381 |
| .11 | .9970 | .05491 | .00151 | .00003 | 0 |
| .045 | .9995 | .02249 | .00025 | 0 | 0 |

The calculations for this operation were performed by hand, and the results are listed on a table in appendix IV.

The total power contributed by those voltage components whose magnitudes are greater than .01 is 99.85% and hence, the spectrum is well analyzed by considering only these terms. The spectrum of this modulation is shown in illustration #10. Only the positive frequency terms greater in magnitude than the carrier have been shown.

$B_{.99} = \pm$ 200,000 cps.

Since the highest frequency term is 300,000 cycles, the carrier-to-noise ratio at the input is

$$S/N \text{ (db)} = 10 \log \frac{S_c}{2\eta B}$$

$$= 10 \log \left\{ \frac{.5}{2(.9875 \times 10^{-8})(600{,}000)} = 10 \log 42.2 \right.$$

$$= 16.24 \text{ db}$$

$$S_o/N_o = 3\,(\Phi_{veff})^2 \frac{S_c}{2\eta f_m}$$

$$= 3\,(.99)^2 \frac{.495}{2(.9875 \times 10^{-8})(20{,}000)} = 3{,}680$$

The last case to be analyzed is for PM of the carrier with six tones whose relative amplitudes are the same as case number four. The phase deviation index for the 100 KC term is $\phi_v = 2.18$

$$f_s(t) = E_c \sum_{i=-\infty}^{\infty} \sum_{j=-\infty}^{\infty} \sum_{k=-\infty}^{\infty} \sum_{l=-\infty}^{\infty} \sum_{m=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} J_i(.1)$$

$$J_j(.1)\, J_k(.1)\, J_l(.1)\, J_m(.24)\, J_n(2.18)$$

$$\sin(\omega_c + i\omega_1 + j\omega_2 + k\omega_3 + l\omega_4 + m\omega_5 + n\omega_6)t$$

The Bessel functions values used in this analysis are listed below:

| X | $J_0$ | $J_1$ | $J_2$ | $J_3$ | $J_4$ | $J_5$ |
|---|---|---|---|---|---|---|
| 2.18 | .1215 | .5587 | .3911 | .1589 | .04541 | .01048 |
| .24 | .9857 | .1191 | .007165 | .00028 | 0 | |
| .1 | .9975 | .04994 | .001249 | .00002 | 0 | |

The calculations for this operation were performed by hand, and the results are listed in appendix V.

The total power contributed by those voltage components whose magnitudes are greater than .01 is 98.96%, and hence the spectrum is well

analyzed by considering only these terms. The spectrum of this modulation is shown in illustration #11. Only the positive frequency terms greater in magnitude than the carrier are shown.

Ninety-nine percent of the power is contained in an i-f bandwidth of $B_{.99} = \pm 400{,}000$ cycles

$$S/N\ (db) = 10 \log \frac{S}{2\eta B} = 10 \log \frac{.4947}{2(.9875 \times 10^{-8})(.8 \times 10^{6})}$$

$$= 15 \text{ db}$$

$$\phi_{veff} = \frac{\phi_v}{B_{.99}} B_{15\ db} = \frac{2.18\,(\pm 400{,}000)}{\pm 400{,}000} = 2.18$$

$$\frac{S_o}{N_o} = 3\left(\phi_{veff\ 100\,kc}\right)^2 \frac{S_c}{2\eta f_m}$$

$$= 3(2.18)^2 \frac{.495}{2(.9875 \times 10^{-8})(2 \times 10^{4})} = 17{,}900$$

## 5. Conclusion

In comparing AM-SSB versus PM, three basic things have to be considered.

(1) The bandwidth should be kept as narrow as possible.

(2) The $S_o/N_o$ ratio should be as high as possible.

(3) The signal should send as much information as possible.

The following letters will identify the different forms of modulations considered:

A -- AM-SSB; Six Tones of Equal Amplitude; Tones--32 cps, 160 cps, 800 cps, 4 KC, 20 KC, and 100 KC.

B -- AM-SSB; Six Tones of Equal Amplitude; Tones--80 KC, 96 KC, 99.2 KC, 99.84 KC, 99.968 KC, and 100 KC.

C -- PM; Six Tones of Equal Amplitude; Tones--80 KC, 96 KC, 99.2 KC, 99.84 KC, 99.968 KC, and 100 KC; $\phi_v = .7$

D -- PM; Six Tones of Equal Amplitude; Tones--80 KC, 96 KC, 99.2 KC, 99.84 KC, 99.968 KC, and 100 KC; $\phi_v = 1.0$

E -- PM; Six Tones of Equal Amplitude; Tones--80 KC, 96 KC, 99.2 KC, 99.84 KC, 99.968 KC, and 100 KC; $\phi_v = 1.5$

F -- PM; Six Tones; 100 KC tone heavily weighted; Tones--80 KC, 96 KC, 99.2 KC, 99.84 KC, 99.968 KC, and 100 KC; $\phi_v$ for 100 KC tone = .99

G -- PM; Six Tones; 100 KC tone heavily weighted; Tones--80 KC, 96 KC, 99.2 KC, 99.84 KC, 99.968 KC, and 100 KC; $\phi_v$ for 100 KC tone = 2.18.

A table of the different forms of modulation listed according to increasing bandwidth is listed below:

Modulation Technique versus Bandwidth

| Order | Form of Modulation | Bandwidth |
|---|---|---|
| 1 | B | 20,000 cps. |
| 2 | A | 100,000 cps. |
| 3 | F | 400,000 cps. |
| 4 | C | 540,000 cps. |
| 5 | E | 550,000 cps. |
| 6 | D | 800,000 cps. |
| 6 | G | 800,000 cps. |

A table of the different forms of modulation listed according to decreasing signal-to-noise ratio is shown below:

$S_o/N_o$ versus Modulation Technique

| Order | Form of Modulation | $S_o'/N_o$ | Db improvement over Form A |
|---|---|---|---|
| 1 | G | 17,900 | 18.5 db |
| 2 | F | 3,680 | 11.6 db |
| 3 | D | 3,675 | 11.6 db |
| 4 | C | 1,845 | 8.63 db |
| 5 | B | 1,265 | 7.0 db |
| 6 | E | 920 | 5.6 db |
| 7 | A | 253 | ------ |

Modulation forms A, B, C, D, and E carry more information than forms F and G.

Other than bandwidth, PM is superior to AM-SSB modulation. PM receivers also do not have the difficult AGC problem, which is inherent in AM modulation systems where the signals are so dynamic; as is the case in radio tracking of space vehicles.

Modulation form "G" is recommended as a good form of modulation to use in high-accuracy space vehicle tracking.

When a carrier is PM or FM modulated by a complex wave, it is recommended that the bandwidth be defined as that which will pass 99% of the power transmitted versus the usually accepted rule, which is based on the magnitude of the voltage component.

For example: a standard table recommends a bandwidth $6\ f_m$ wide for a $\phi_v$ equal to 1.[19] This would be 600,000 cycles wide. From the analysis of modulation form "D", the 99% power bandwidth would equal 800,000 cycles, and similarly for $\phi_v = 1.5$, the table recommends $7\ f_m = 700{,}000$ cycles, while the analysis of modulation form "E" showed that the bandwidth should be 1,350,000 cycles wide.

[19] M. Schwartz, op. cit., p. 129.

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APPENDIX I

FORTRAN PROGRAM #1

```
      PROGRAM IONE5
      DIMENSION A(20), X(20)
      SUMS = 0.
      A(1) = 0.
      A(2) = 1.
      A(3) = -1.
      A(4) = 2.
      A(5) = -2.
      A(6) = 3.
      A(7) = -3.
      A(8) = 4.
      A(9) = -4.
      A(10) = 5.
      A(11) = -5.
      X(1) = .5118
      X(2) = .5579
      X(3) = .5579
      X(4) = .2321
      X(5) = .2321
      X(6) = .06096
      X(7) = .06096
   10 DO100I = 1,7
      XI = X(I)
      AI = A(I)
      DO 100 J = 1,7
      XJ = X(J)
      AJ = A(J)
      DO 100 K = 1,7
      XK = X(K)
      AK = A(K)
      DO 100 L = 1,7
      XL = X(L)
      AL = A(L)
      DO 100 M = 1,7
      XM = X(M)
      AM = A(M)
      DO 100 N = 1,7
      XN = X(N)
      AN = A(N)
      AMAG = XI * XJ * XK * XL * XM * XN
      IF (AMAG - .001) 100,50,50
   50 SAMAG = AMAG * AMAG
      SUMS = SUMS + SAMAG
      FREQ = AI *80000.+AJ*96000.+AK*99200.+AL*99840.+AM*99968.+AN*100000.
   60 FORMAT (6F5.0,2F16.11, F13.1)
      WRITE OUTPUT TAPE 4,60,AI,AJ,AK,AL,AM,AN,AMAG,SAMAG,FREQ
```

```
100   CONTINUE
61    FORMAT (1HOE20.10)
      SUMS = SUMS / 2.
      WRITE OUTPUT TAPE 4, 61, SUMS
      END
      END
```

The values of X(1) through X(7) are for a phase deviation index of 1.5. For the other analyses, the appropriate Bessel function values were inserted.

| | $\phi_v = 1.0$ | $\phi_v = 0.7$ |
|---|---|---|
| X(1) | .7652 | .8812 |
| X(2) | .4401 | .3290 |
| X(3) | .4401 | .3290 |
| X(4) | .1149 | .0588 |
| X(5) | .1149 | .0588 |
| X(6) | .0196 | .0069 |
| X(7) | .0196 | .0069 |

APPENDIX II

FORTRAN PROGRAM #2

```
      PROGRAM IONE5P
      DIMENSION X(20)
      SUMS = 0
      SUMSA = 0
      SUMSB = 0
      SUMSC = 0
      SUMSD = 0
      TUMS = 0
      TUMSA = 0
      TUMSB = 0
      TUMSC = 0
      TUMSD = 0
      X(1) = .5118
      X(2) = .5579
      X(3) = .5579
      X(4) = .2321
      X(5) = .2321
      X(6) = .06096
      X(7) = .06096
10    DO100I = 1,7
      XI = X(I)
      DO100J = 1,7
      XJ = X(J)
      DO100K = 1,7
      XK = X(K)
      DO100L = 1,7
      XL = X(L)
      DO100M = 1,7
      XM = X(M)
      DO100N = 1,7
      XN = X(N)
      AMAG = XI*XJ*XK*XL*XM*XN
      SMAG = AMAG*AMAG
      IF (SMAG-.0001)30,20,20
20    SUMS = SUMS + SMAG
      TUMS = TUMS + 1.
      GO TO 100
30    IF(SMAG-.000025)50,40,40
40    SUMSA = SUMSA + SMAG
      TUMSA = TUMSA + 1.
      GO TO 100
```

```
50    IF(SMAG-.000001)65,60,60
60    SUMSB = SUMSB + SMAG
      TUMSB = TUMSB + 1.
      GO TO 100
65    IF(SMAG -.00000025)75,70,70
70    SUMSC = SUMSC + SMAG
      TUMSC = TUMSC + 1.
      GO TO 100
75    IF(SMAG-.00000001)100,90,90
90    SUMSD = SUMSD + SMAG
      TUMSD = TUMSD + 1.
100   CONTINUE
110   FORMAT (5(F14.10,F10.0))
      PRINT110,SUMS,TUMS,SUMSA,TUMSA,SUMSB,TUMSB,SUMSC,TUMSC,SUMSD,TUMSD
      END
      END
```

## APPENDIX III

### FORTRAN PROGRAM #3

```
      PROGRAM IONE5PBW
      DIMENSION A(20), X(20)
      SUMS = 0
      SUMSA = 0
      SUMSB = 0
      SUMSC = 0
      SUMSD = 0
      SUMSE = 0
      SUMSG = 0
      SUMSH = 0
      A(1) = 0.
      A(2) = 1.
      A(3) = -1.
      A(4) = 2.
      A(5) = -2.
      A(6) = 3.
      A(7) = -3.
      A(8) = 4.
      A(9) = -4.
      A(10) = 5.
      A(11) = -5.
      X(1) = .5118
      X(2) = .5579
      X(3) = .5579
      X(4) = .2321
      X(5) = .2321
      X(6) = .06096
      X(7) = .06096
10    DO100I = 1,7
      XI = X(I)
      AI = A(I)
      DO100J = 1,7
      XJ = X(J)
      AJ = A(J)
      DO100K = 1,7
      XK = X(K)
      AK = A(K)
      DO100L = 1,7
      XL = X(L)
      AL = A(L)
      DO100M = 1,7
      XM = X(M)
      AM = A(M)
      DO 100 N = 1,7
      XN = X(N)
      AN = A(N)
```

```
      AMAG = XI*XJ*XK*XL*XM*XN
      IF(AMAG-.0001) 100,15,15
15    SMAG = AMAG*AMAG
      FREQ = AI*80000.+AJ*96000.+AK*99200.+AL*99840.+AM*99968.+AN*100000.
      IF(FREQ) 100,20,20
20    IF(175000.-FREQ) 40,30,30
30    SUMS = SUMS+SMAG
      GO TO 100
40    IF(275000.-FREQ)50,45,45
45    SUMSA = SUMSA+SMAG
      GO TO 100
50    IF(375000.-FREQ) 60,55,55
55    SUMSB = SUMSB + SMAG
      GO TO 100
60    IF(475000.-FREQ)70,65,65
65    SUMSC = SUMSC + SMAG
      GO TO 100
70    IF(575000.-FREQ)75,72,72
72    SUMSD = SUMSD + SMAG
      GO TO 100
75    IF(675000.-FREQ)80,77,77
77    SUMSE = SUMSE + SMAG
      GO TO 100
80    IF(775000.-FREQ)85,82,82
82    SUMSG = SUMSG +SMAG
      GO TO 100
85    IF(875000.-FREQ)100,90,90
90    SUMSH = SUMSH + SMAG
100   CONTINUE
110   FORMAT (8F15.10)
      PRINT110,SUMS,SUMSA,SUMSB,SUMSC,SUMSD,SUMSE,SUMSG,SUMSH
      END
      END
```

## APPENDIX IV

Calculations for a PM wave with the modulating tones weighted and $\phi_v = .99$ for the 100 KC term

| i | j | k | l | m | n | Magnitude of Voltage Term | Squared | Frequency from Carrier |
|---|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 | .7658 | .5864 | 0 |
| 0 | 0 | 0 | 0 | 0 | 1 | .4346 | .1889 | 100,000 |
| 0 | 0 | 0 | 0 | 0 | -1 | .4346 | .1889 | -100,000 |
| 0 | 0 | 0 | 0 | 0 | 2 | .1122 | .01259 | 200,000 |
| 0 | 0 | 0 | 0 | 0 | -2 | .1122 | .01259 | -200,000 |
| 0 | 0 | 0 | 0 | 0 | 3 | .0189 | .0003572 | 300,000 |
| 0 | 0 | 0 | 0 | 0 | -3 | .0189 | .0003572 | -300,000 |
| 0 | 0 | 0 | 0 | 1 | 0 | .04217 | .0017783 | 99,968 |
| 0 | 0 | 0 | 0 | -1 | 0 | .04217 | .0017783 | -99,968 |
| 0 | 0 | 0 | 0 | 1 | 1 | .02394 | .0005731 | 199,968 |
| 0 | 0 | 0 | 0 | -1 | 1 | .02394 | .0005731 | 32 |
| 0 | 0 | 0 | 0 | 1 | -1 | .02394 | .0005731 | -32 |
| 0 | 0 | 0 | 0 | -1 | -1 | .02394 | .0005731 | -199,968 |
| 0 | 0 | 0 | 1 | 0 | 0 | .01723 | .0002969 | 99,680 |
| 0 | 0 | 0 | -1 | 0 | 0 | .01723 | .0002969 | -99,680 |
| 0 | 0 | 1 | 0 | 0 | 0 | .01723 | .0002969 | 99,200 |
| 0 | 0 | -1 | 0 | 0 | 0 | .01723 | .0002969 | -99,200 |
| 0 | 1 | 0 | 0 | 0 | 0 | .01723 | .0002969 | 96,000 |
| 0 | -1 | 0 | 0 | 0 | 0 | .01723 | .0002969 | -96,000 |
| 1 | 0 | 0 | 0 | 0 | 0 | .01723 | .0002969 | 80,000 |
| -1 | 0 | 0 | 0 | 0 | 0 | .01723 | .0002969 | -80,000 |

$$S_c = .2932 + .1889 + .01259 + .0003572 + .001778 + .001462 + .001188 = .4992$$

APPENDIX V

Calculations for a PM Wave with the modulating tones weighted and $\phi_v$ = 2.18 for the 100 KC term

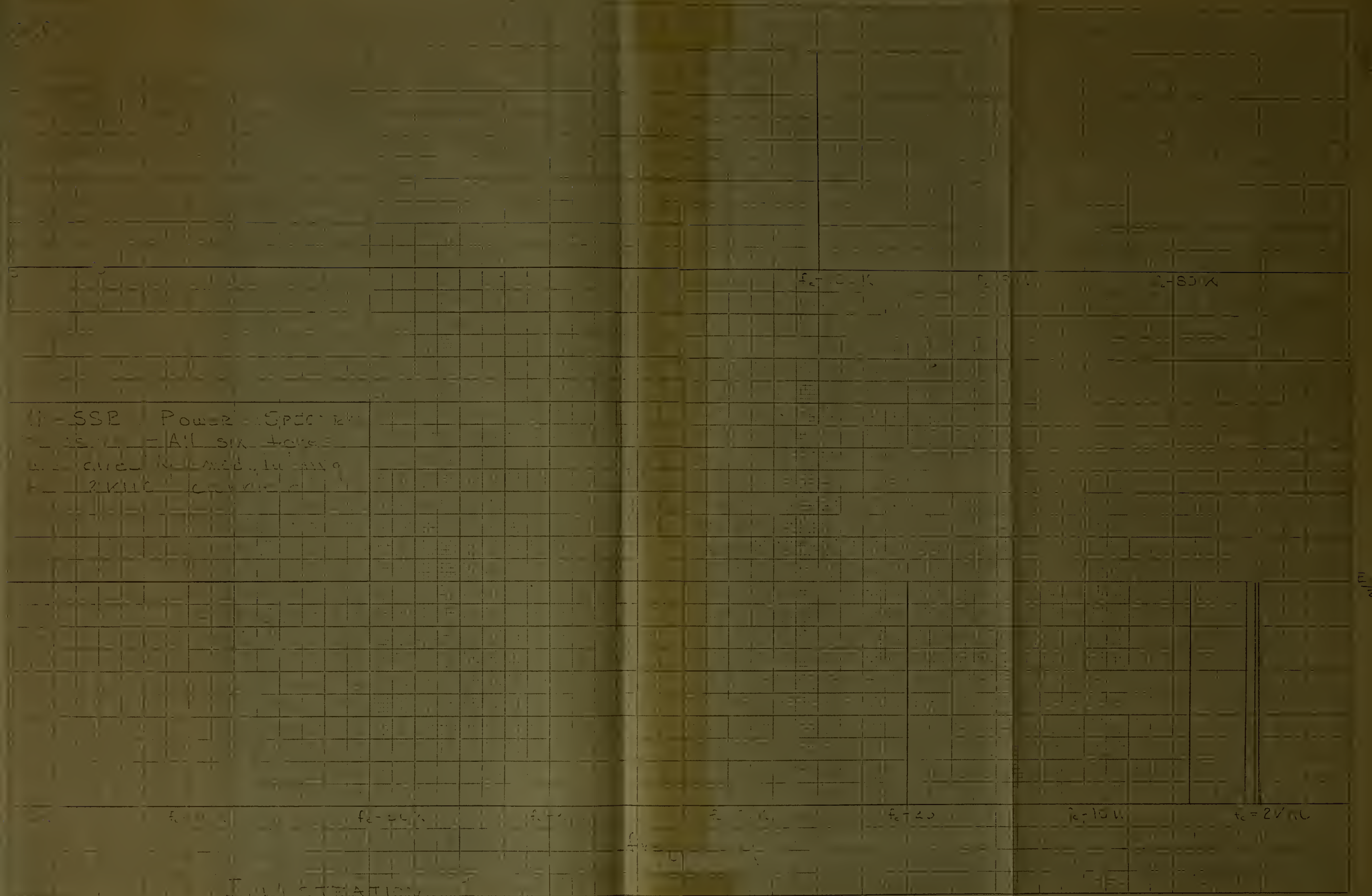
| i | j | k | l | m | n | Magnitude of Voltage Terms | Squared | Frequency from Carrier |
|---|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 | .1186 | .01407 | 0 |
| 0 | 0 | 0 | 0 | 0 | 1 | .5452 | .2972 | 100,000 |
| 0 | 0 | 0 | 0 | 0 | -1 | .5452 | .2972 | -100,000 |
| 0 | 0 | 0 | 0 | 0 | 2 | .3816 | .1456 | 200,000 |
| 0 | 0 | 0 | 0 | 0 | -2 | .3816 | .1456 | -200,000 |
| 0 | 0 | 0 | 0 | 0 | 3 | .1551 | .02405 | 300,000 |
| 0 | 0 | 0 | 0 | 0 | -3 | .1551 | .02405 | -300,000 |
| 0 | 0 | 0 | 0 | 0 | 4 | .04431 | .001963 | 400,000 |
| 0 | 0 | 0 | 0 | 0 | -4 | .04431 | .001963 | -400,000 |
| 0 | 0 | 0 | 0 | 0 | 5 | .01023 | .0001046 | 500,000 |
| 0 | 0 | 0 | 0 | 0 | -5 | .01023 | .0001046 | -500,000 |
| 0 | 0 | 0 | 0 | 1 | 0 | .01432 | .0002051 | 99.968 |
| 0 | 0 | 0 | 0 | -1 | 0 | .01432 | .0002051 | -99,968 |
| 0 | 0 | 0 | 0 | 1 | 1 | .06587 | .004339 | 199,968 |
| 0 | 0 | 0 | 0 | -1 | +1 | .06587 | .004339 | 32 |
| 0 | 0 | 0 | 0 | 1 | -1 | .06587 | .004339 | -32 |
| 0 | 0 | 0 | 0 | -1 | -1 | .06587 | .004339 | -199,968 |
| 0 | 0 | 0 | 0 | 1 | 2 | .04611 | .0002126 | 299,968 |
| 0 | 0 | 0 | 0 | -1 | 2 | .04611 | .0002126 | 100,032 |
| 0 | 0 | 0 | 0 | 1 | -2 | .04611 | .0002126 | -100,032 |
| 0 | 0 | 0 | 0 | -1 | -2 | .04611 | .0002126 | -299,968 |
| 0 | 0 | 0 | 0 | 1 | 3 | .01873 | .0003508 | 399,968 |
| 0 | 0 | 0 | 0 | -1 | +3 | .01873 | .0003508 | 200,032 |
| 0 | 0 | 0 | 0 | 1 | -3 | .01873 | .0003508 | -200,032 |
| 0 | 0 | 0 | 0 | -1 | -3 | .01873 | .0003508 | -399,968 |
| 0 | 0 | 0 | 1 | 0 | 1 | .02729 | .0007447 | 199,840 |
| 0 | 0 | 0 | -1 | 0 | 1 | .02729 | .0007447 | 160 |
| 0 | 0 | 0 | 1 | 0 | -1 | .02729 | .0007447 | -160 |
| 0 | 0 | 0 | -1 | 0 | -1 | .02729 | .0007447 | -199,840 |
| 0 | 0 | 1 | 0 | 0 | 1 | .02729 | .0007447 | 199,200 |
| 0 | 0 | -1 | 0 | 0 | 1 | .02729 | .0007447 | 800 |
| 0 | 0 | 1 | 0 | 0 | -1 | .02729 | .0007447 | -800 |
| 0 | 0 | -1 | 0 | 0 | -1 | .02729 | .0007447 | -199,200 |
| 0 | 1 | 0 | 0 | 0 | 1 | .02729 | .0007447 | 196,000 |
| 0 | -1 | 0 | 0 | 0 | 1 | .02729 | .0007447 | 4,000 |
| 0 | 1 | 0 | 0 | 0 | -1 | .02729 | .0007447 | -4,000 |
| 0 | -1 | 0 | 0 | 0 | -1 | .02729 | .0007447 | -196,000 |
| 1 | 0 | 0 | 0 | 0 | 1 | .02729 | .0007447 | 180,000 |
| -1 | 0 | 0 | 0 | 0 | 1 | .02729 | .0007447 | 20,000 |
| 1 | 0 | 0 | 0 | 0 | -1 | .02729 | .0007447 | - 20,000 |
| -1 | 0 | 0 | 0 | 0 | -1 | .02729 | .0007447 | -180,000 |

(Continued)

| i | j | k | l | m | n | Magnitude of Voltage Terms | Squared | Frequency from Carrier |
|---|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 1 | 0 | 2 | .01911 | .0003652 | 299,840 |
| 0 | 0 | 0 | -1 | 0 | 2 | .01911 | .0003652 | 100,160 |
| 0 | 0 | 0 | 1 | 0 | -2 | .01911 | .0003652 | -100,160 |
| 0 | 0 | 0 | -1 | 0 | -2 | .01911 | .0003652 | -299,840 |
| 0 | 0 | 1 | 0 | 0 | 2 | .01911 | .0003652 | 299,200 |
| 0 | 0 | -1 | 0 | 0 | 2 | .01911 | .0003652 | 100,800 |
| 0 | 0 | 1 | 0 | 0 | -2 | .01911 | .0003652 | -100,800 |
| 0 | 0 | -1 | 0 | 0 | -2 | .01911 | .0003652 | -299,200 |
| 0 | 1 | 0 | 0 | 0 | 2 | .01911 | .0003652 | 296,000 |
| 0 | -1 | 0 | 0 | 0 | 2 | .01911 | .0003652 | 104,000 |
| 0 | 1 | 0 | 0 | 0 | -2 | .01911 | .0003652 | -104,000 |
| 0 | -1 | 0 | 0 | 0 | -2 | .01911 | .0003652 | -296,000 |
| 1 | 0 | 0 | 0 | 0 | 2 | .01911 | .0003652 | 280,000 |
| -1 | 0 | 0 | 0 | 0 | 2 | .01911 | .0003652 | 120,000 |
| 1 | 0 | 0 | 0 | 0 | -2 | .01911 | .0003652 | -120,000 |
| -1 | 0 | 0 | 0 | 0 | -2 | .01911 | .0003652 | -280,000 |

$$\begin{aligned}\text{Total Power} = {} & .007 + .2972 + .1456 \\ & + .02405 + .001963 + .0001046 \\ & + .0002051 + .008678 + .0004252 \\ & + .0007016 + .005958 + .002922 = .4948\end{aligned}$$

$$\%\ \text{Total Power} = \frac{.4948}{.5} = 98.96\%$$


SSB Power
All six tones

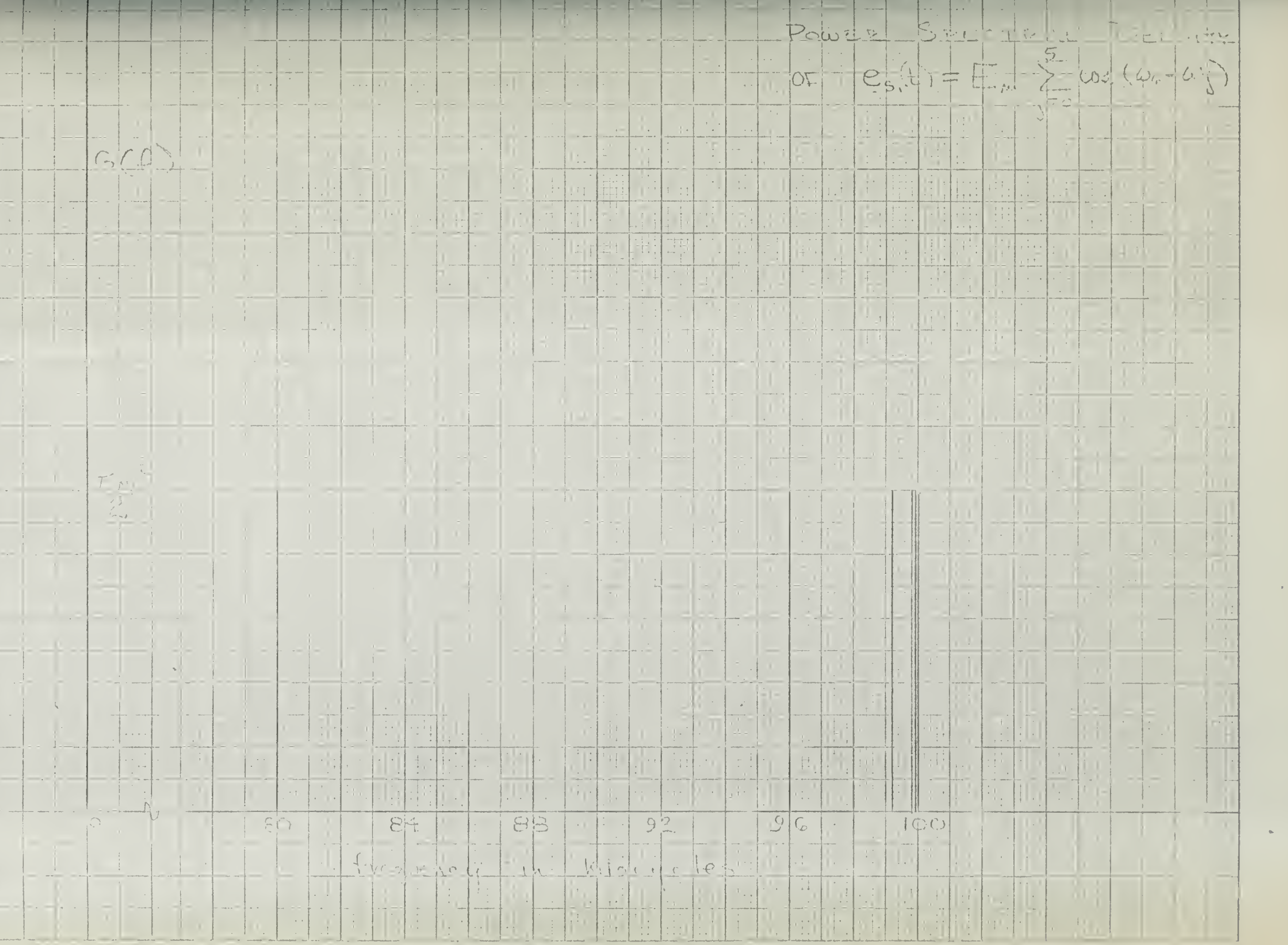
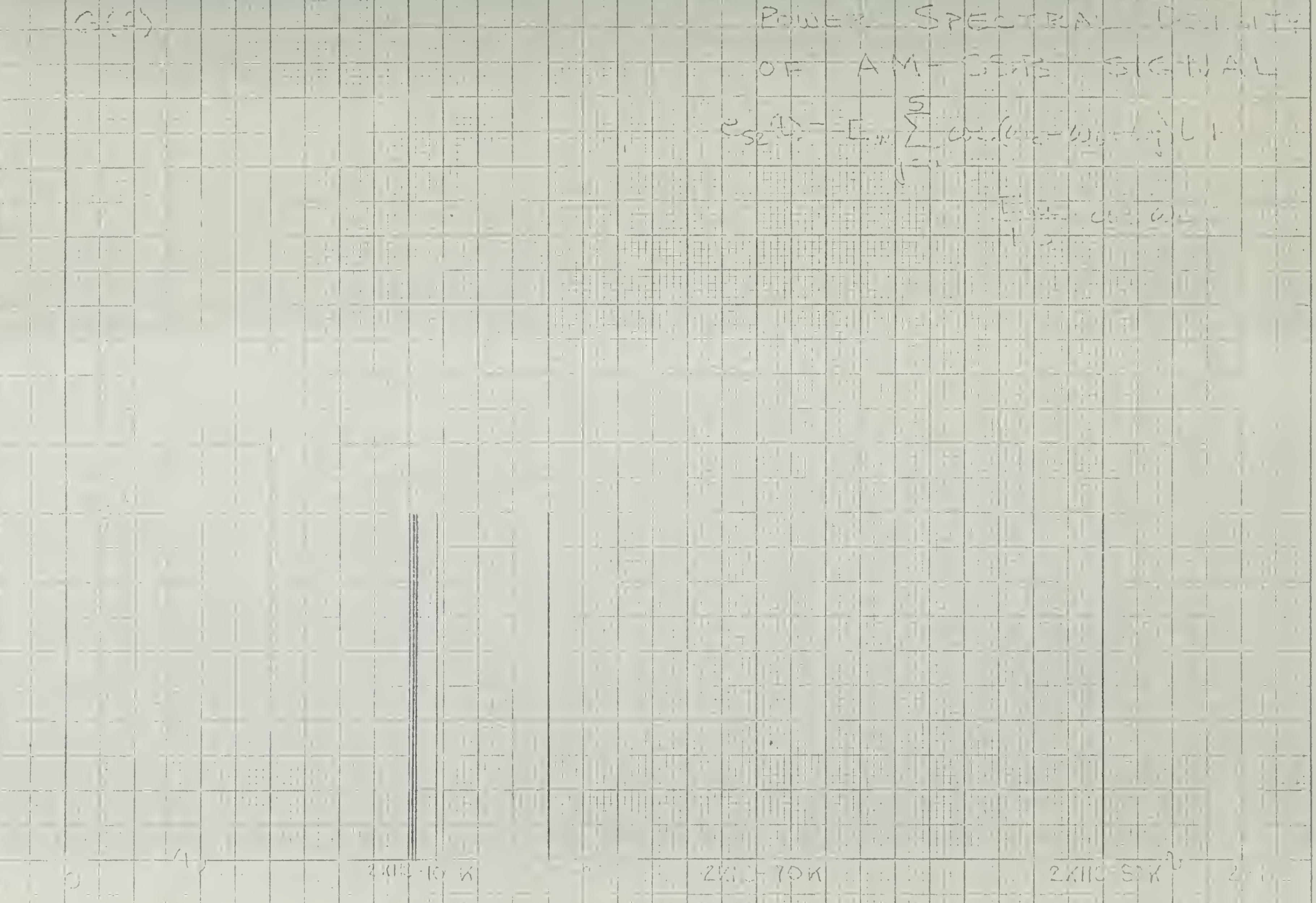



ILLUSTRATION 2


POWER SPECTRAL DENSITY
OF AM-SSB SIGNAL

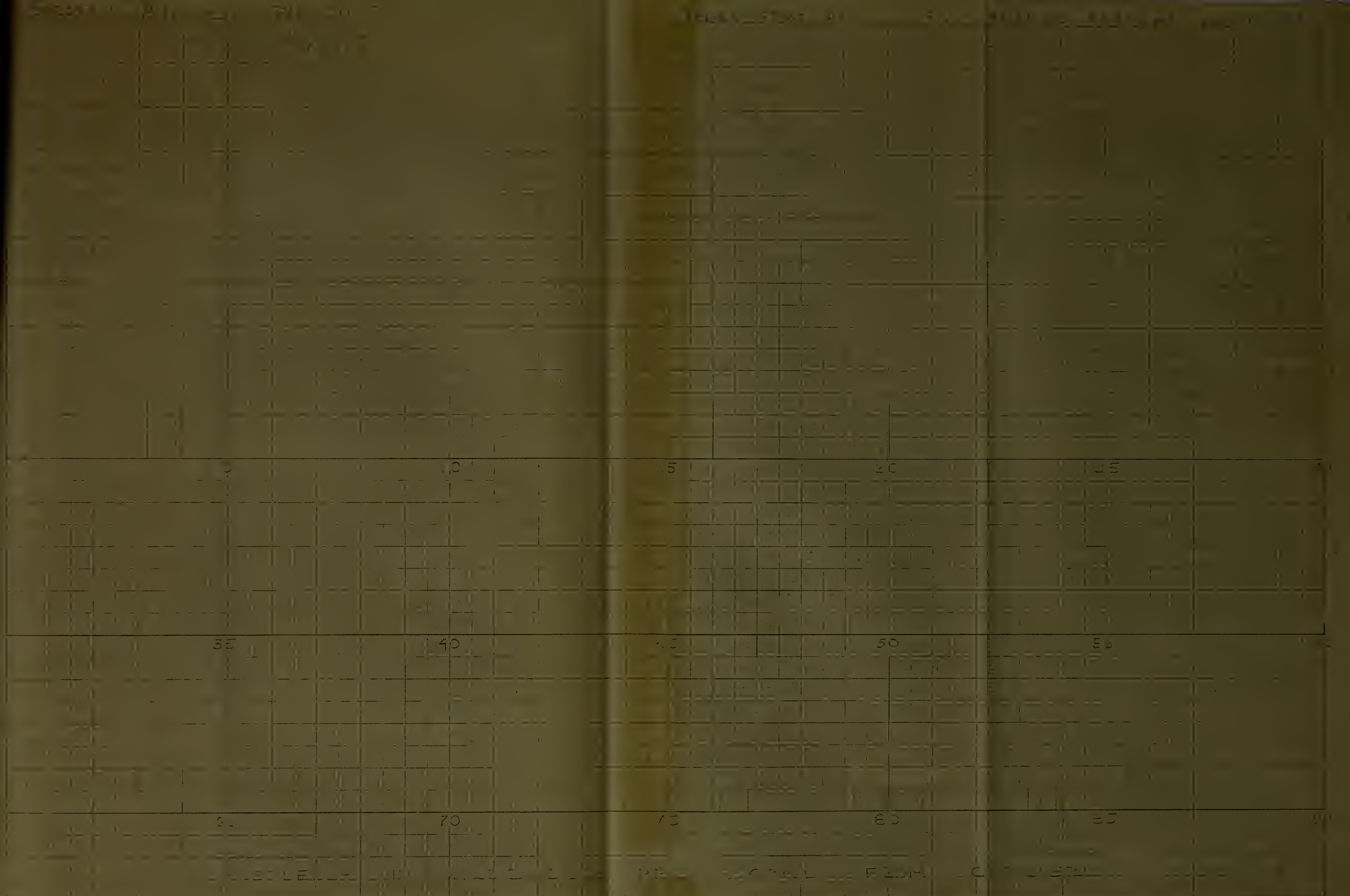
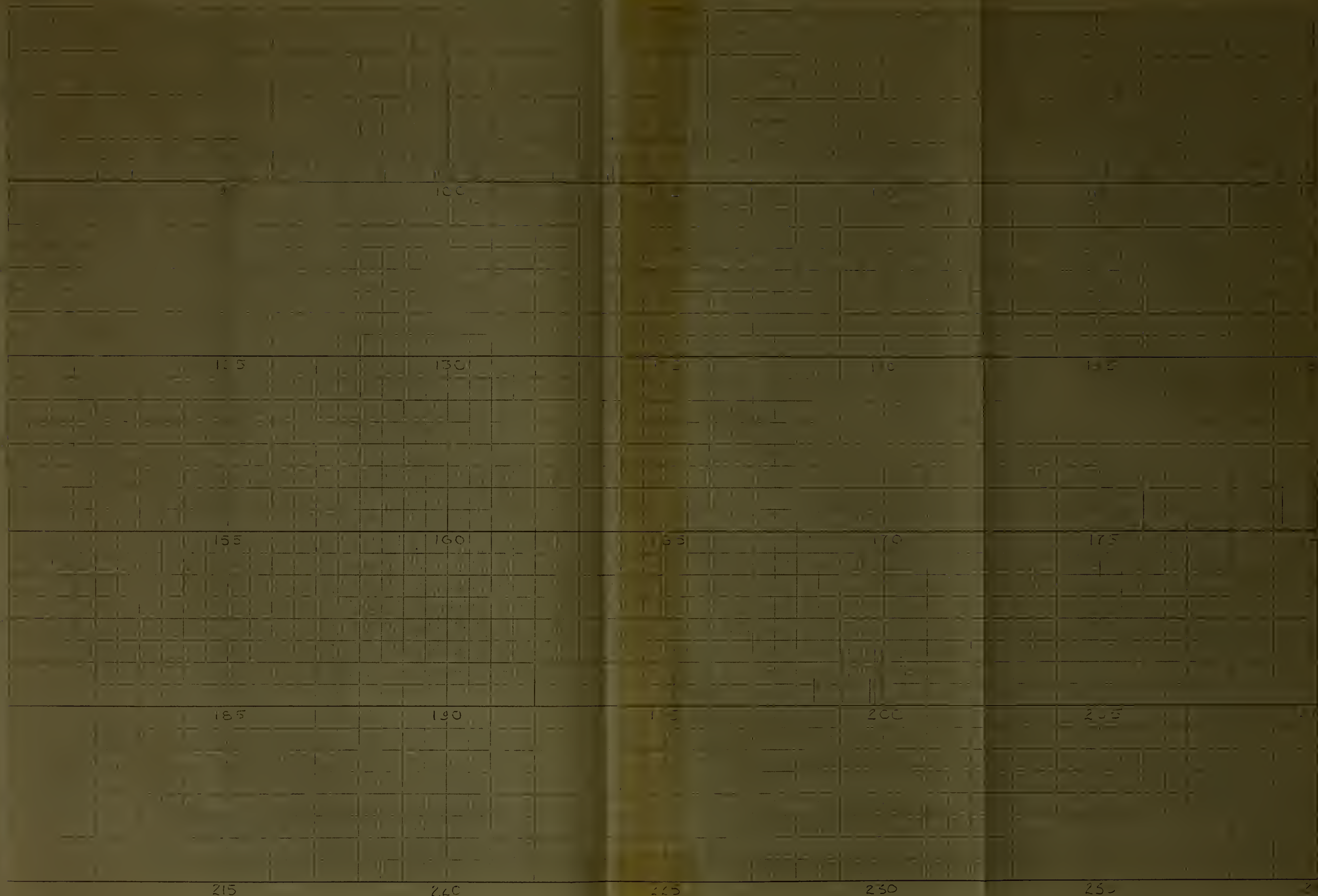
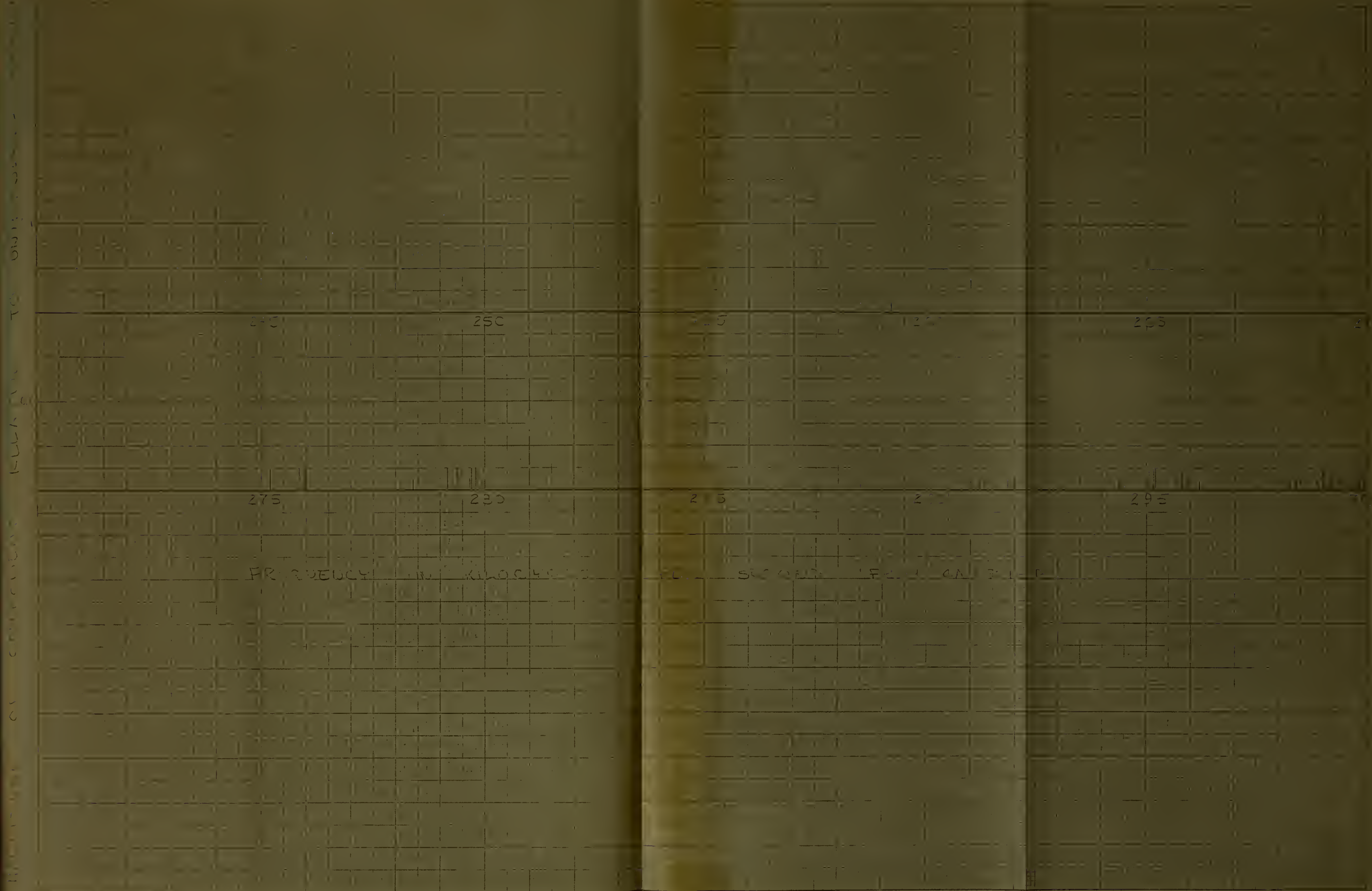
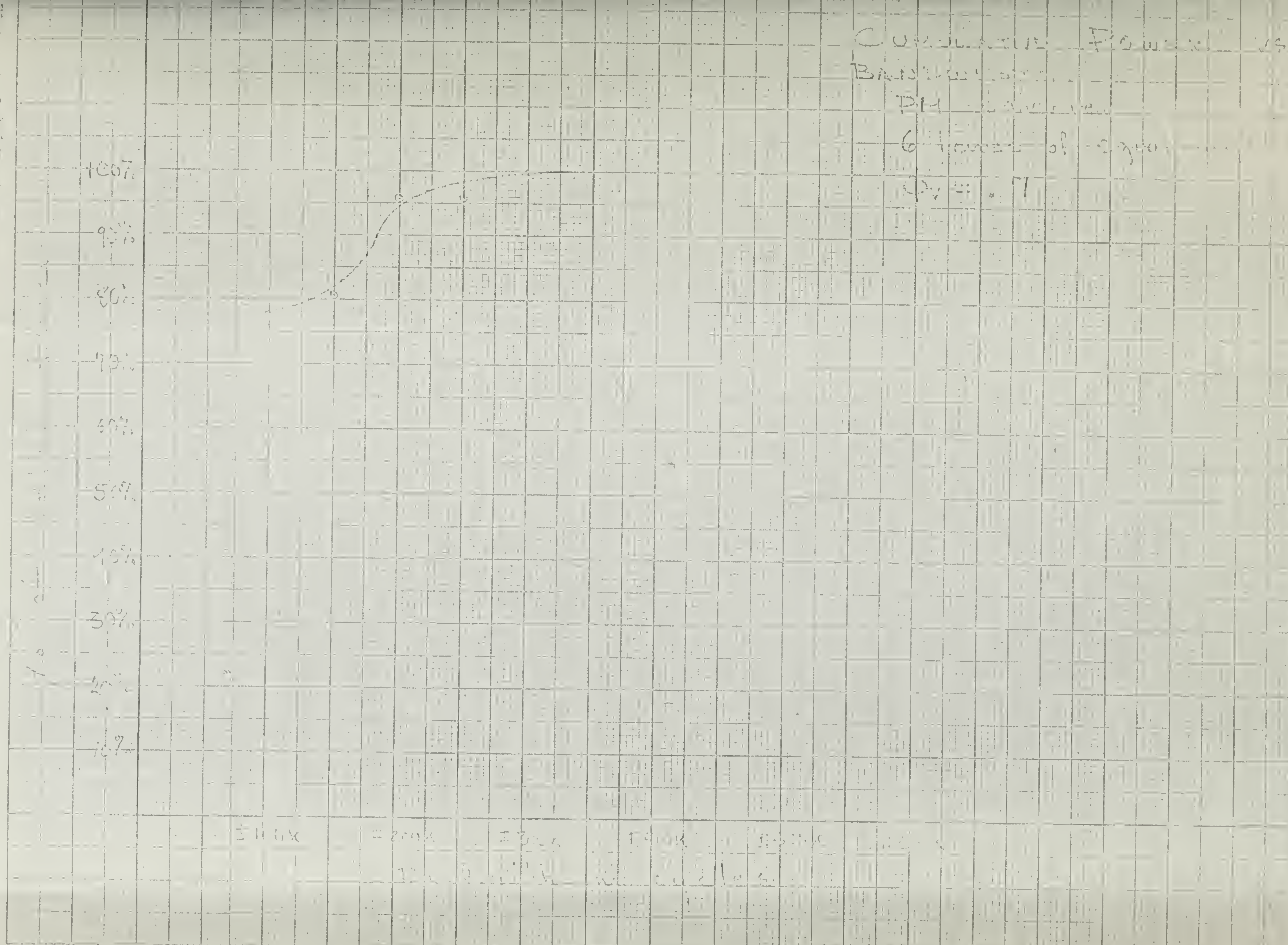


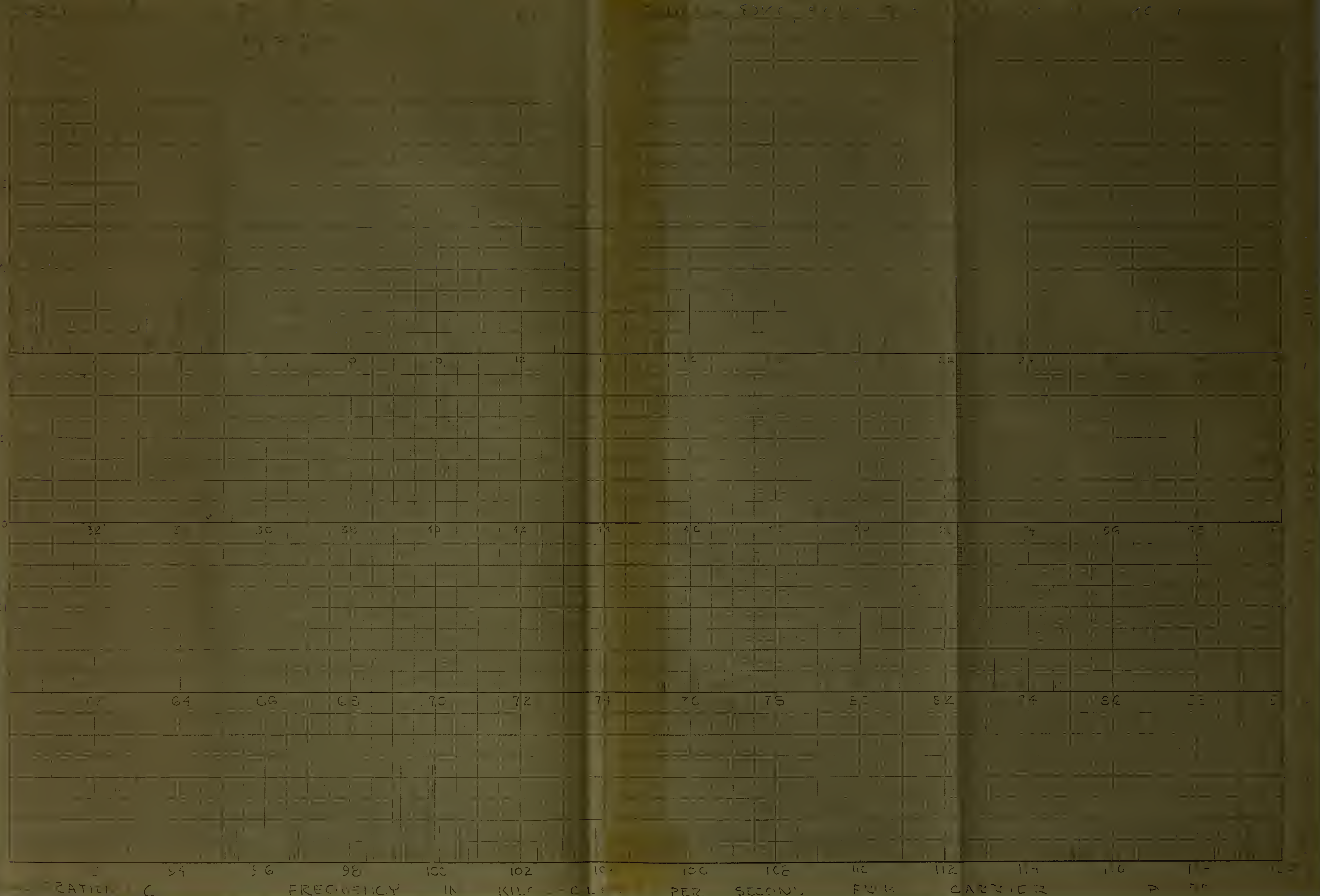

ILLUSTRATION 4

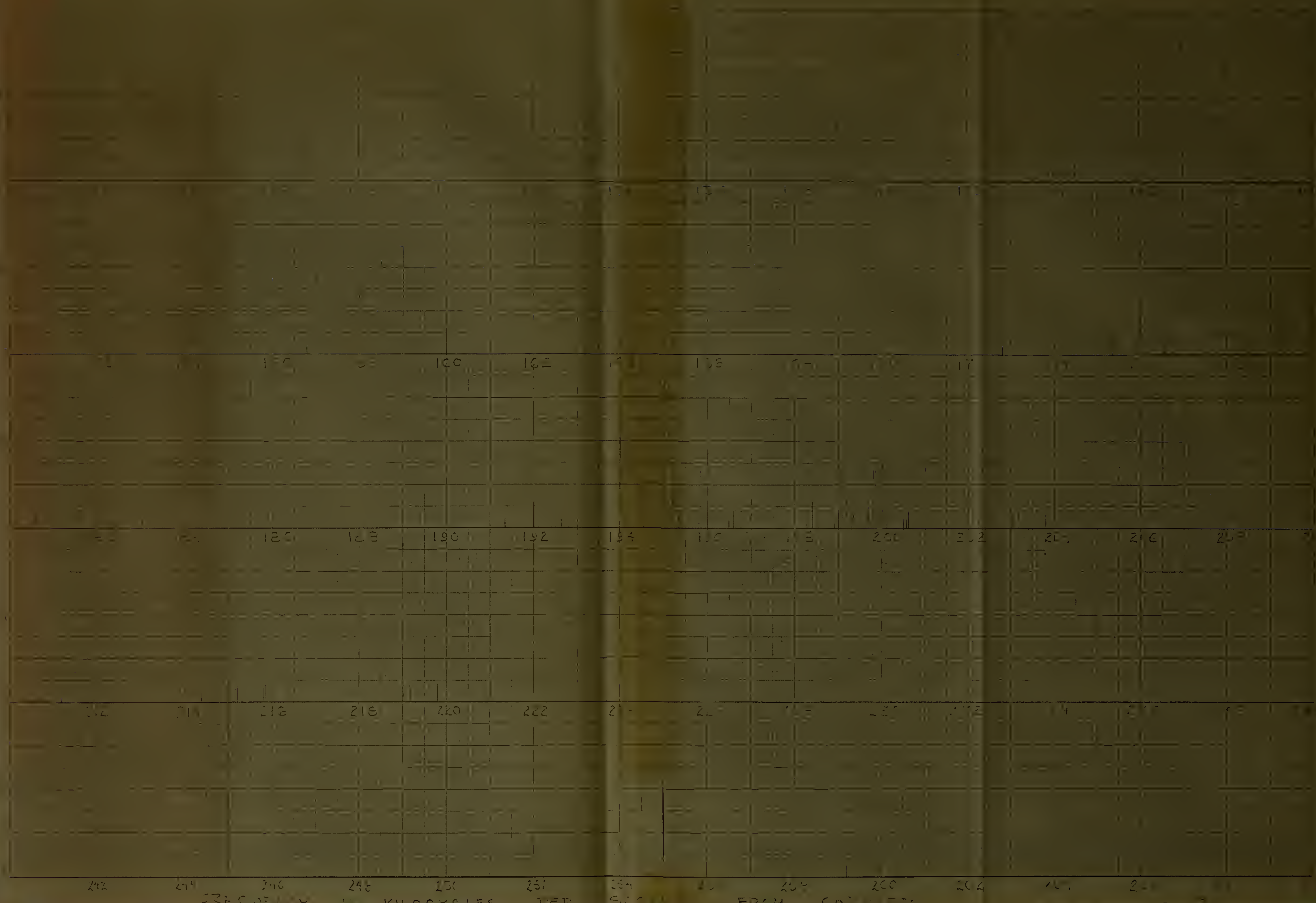



FREQUENCY IN KILOCYCLES PER SECOND FROM CARRIER

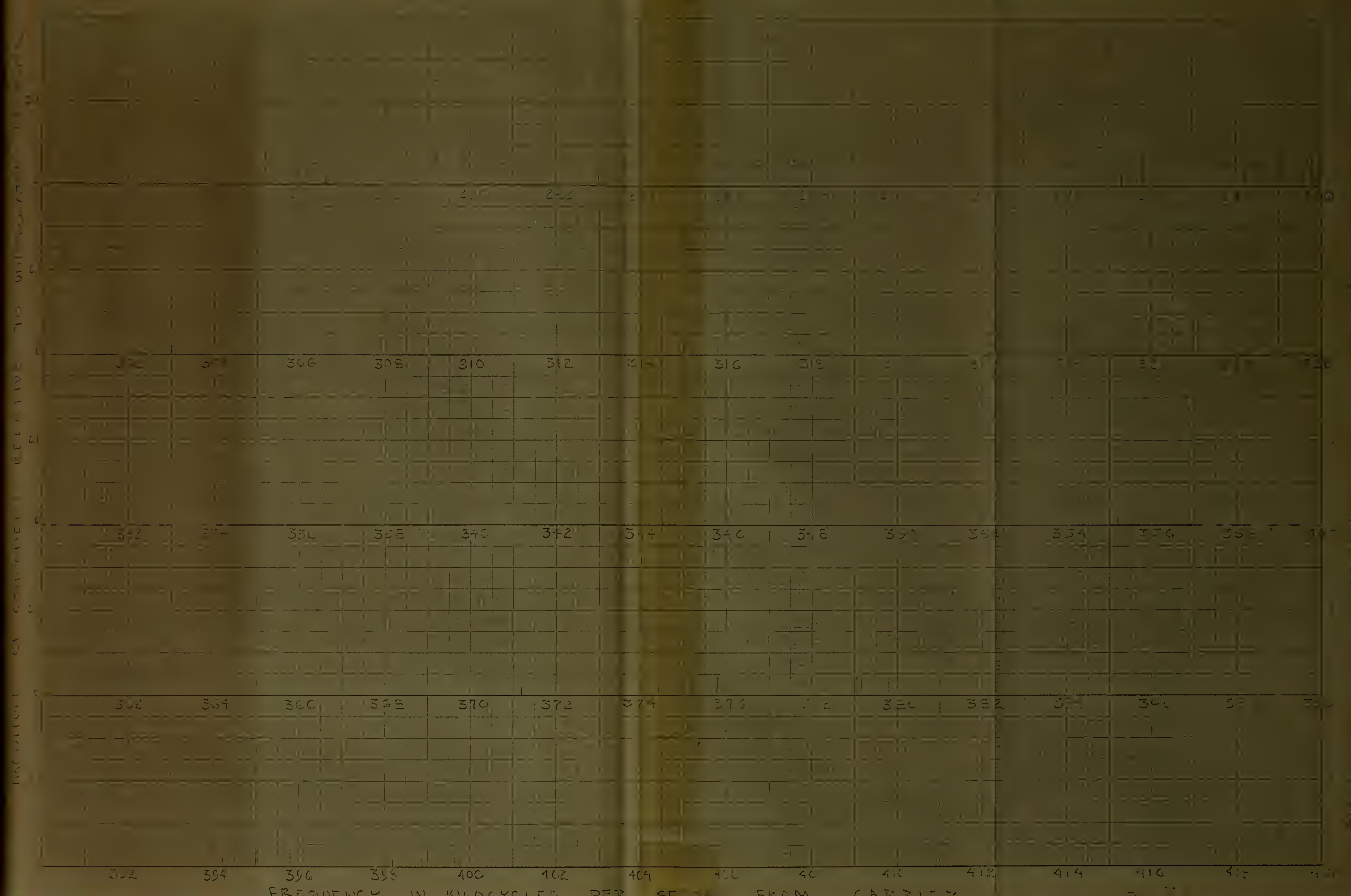

250
275
295


100%
90%
80%
70%
60%
50%
40%
30%
20%
10%


PER



160
162
190
192
194
200
220
222
KILOCYCLES
PER
FROM


306 308 310 312 316
340 342 346 354
362 364 366 370 372 380 382
394 396 400 402 404 412 414 416
FREQUENCY IN KILOCYCLES PER
FROM CARRIER

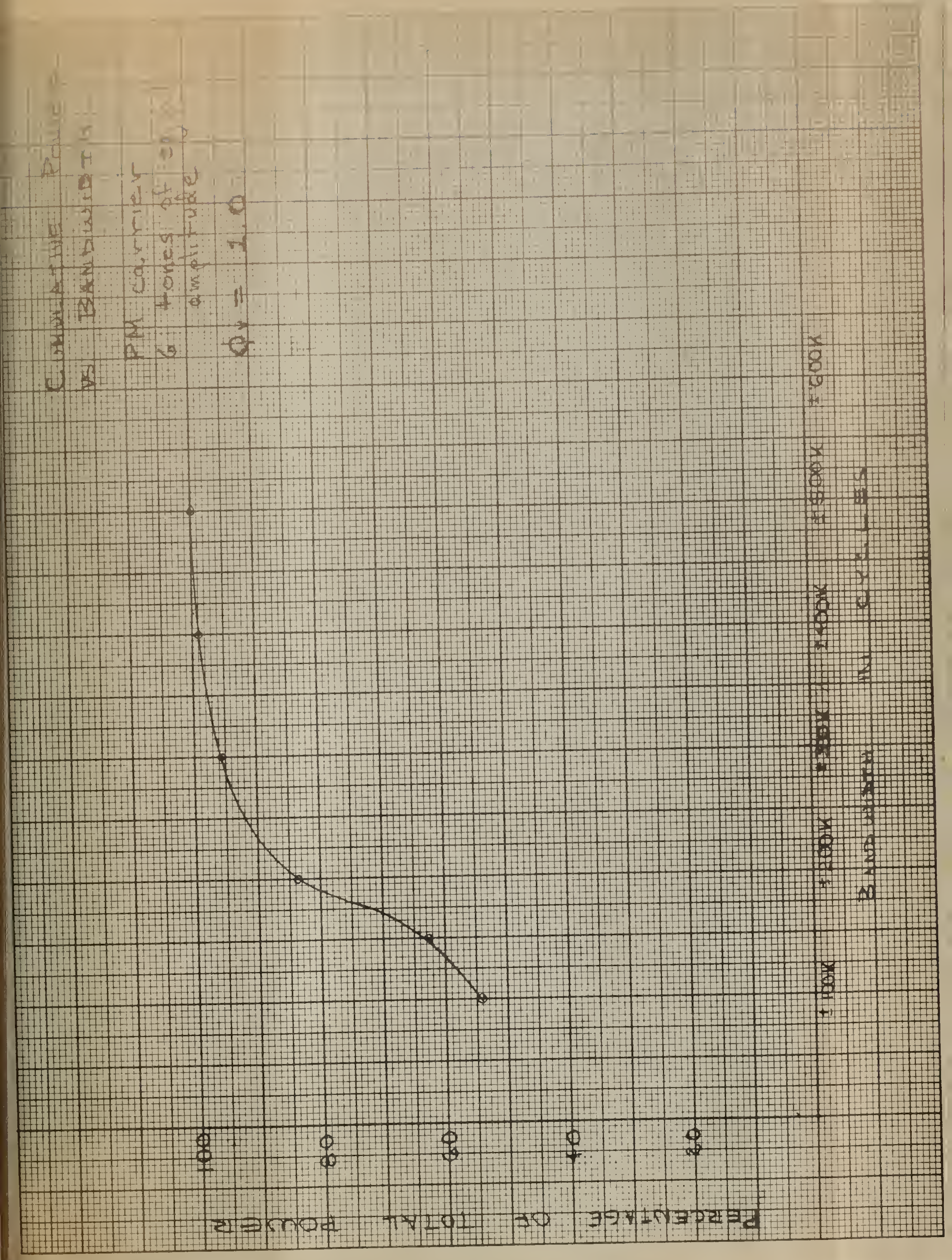
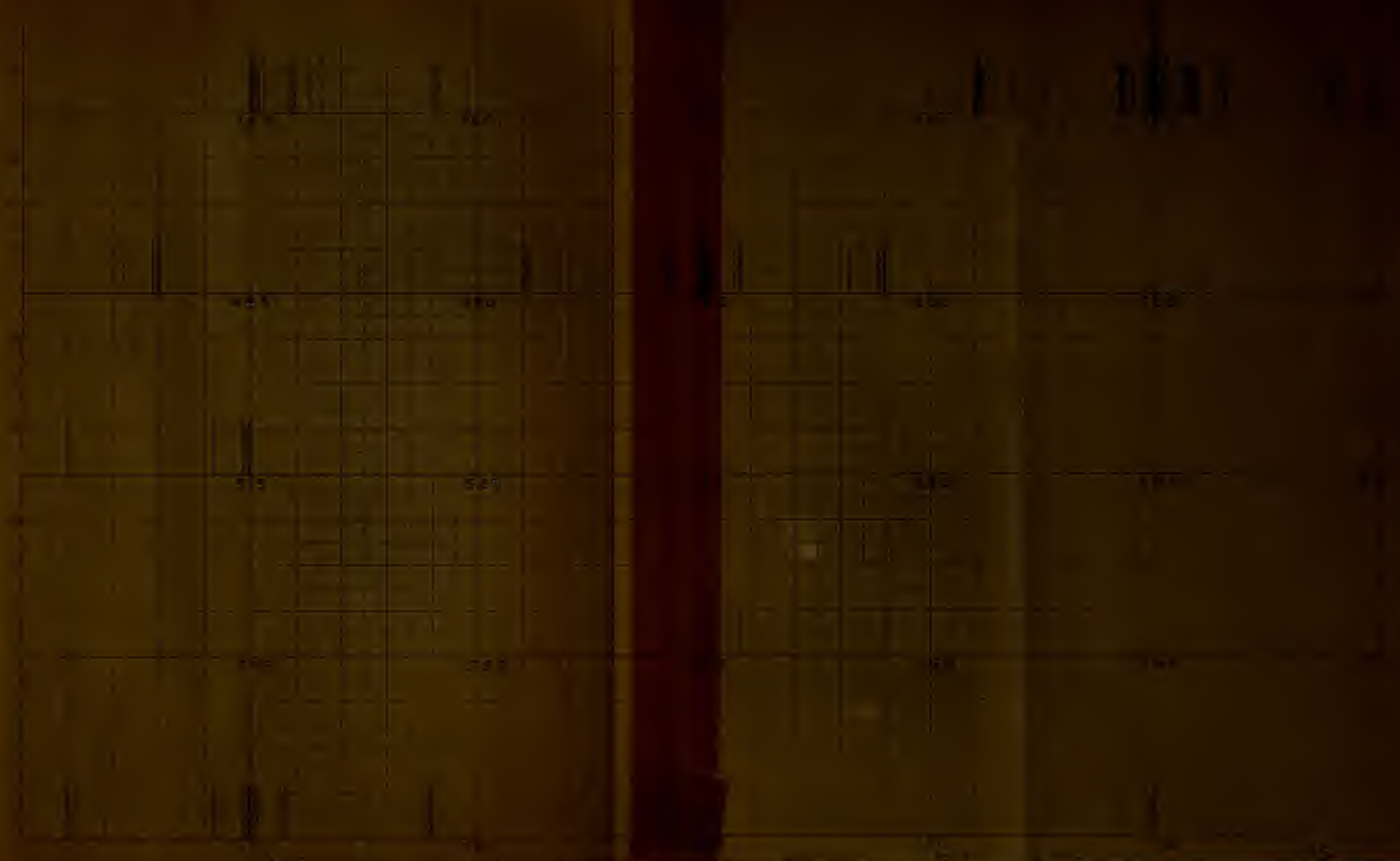
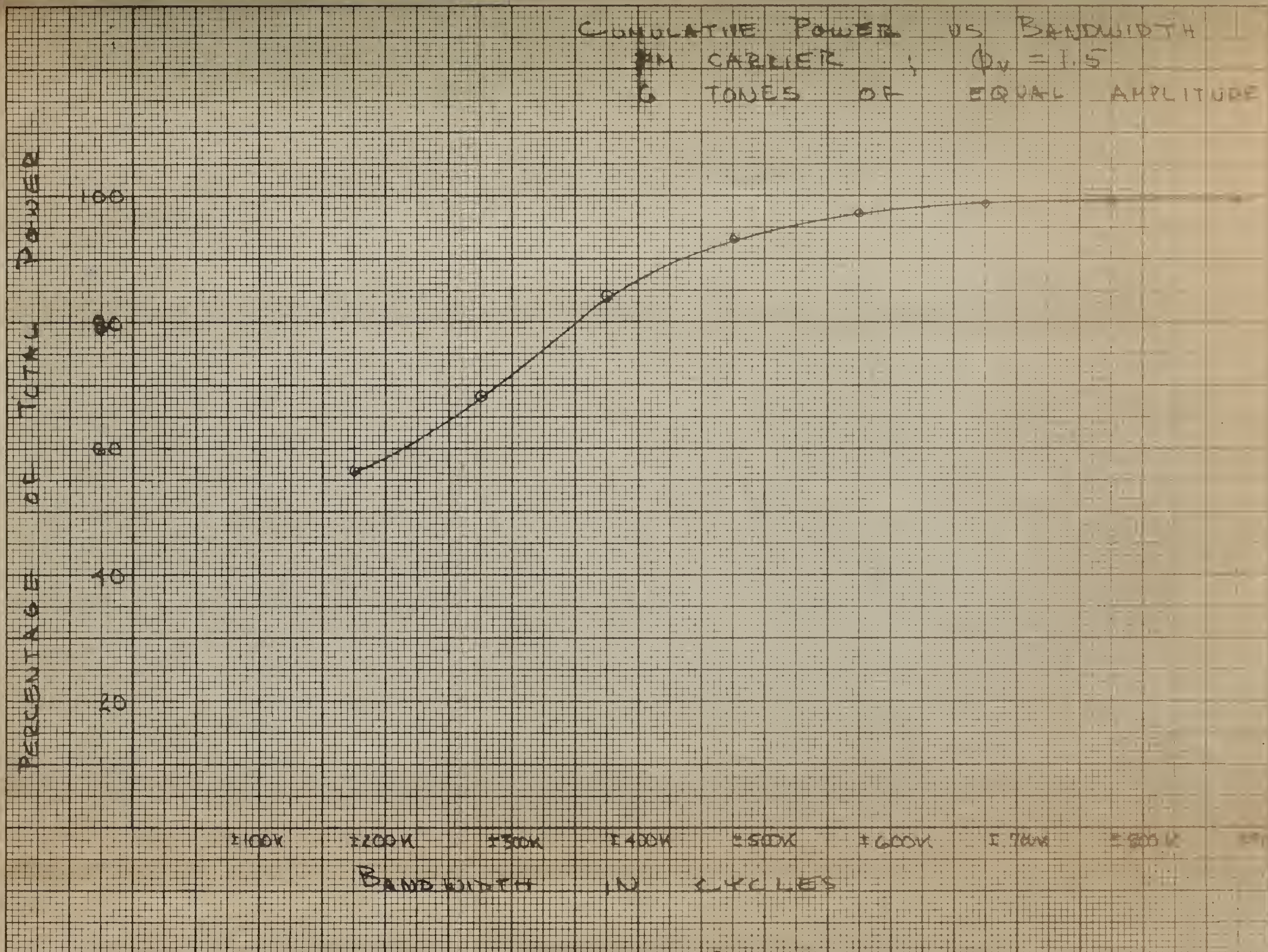


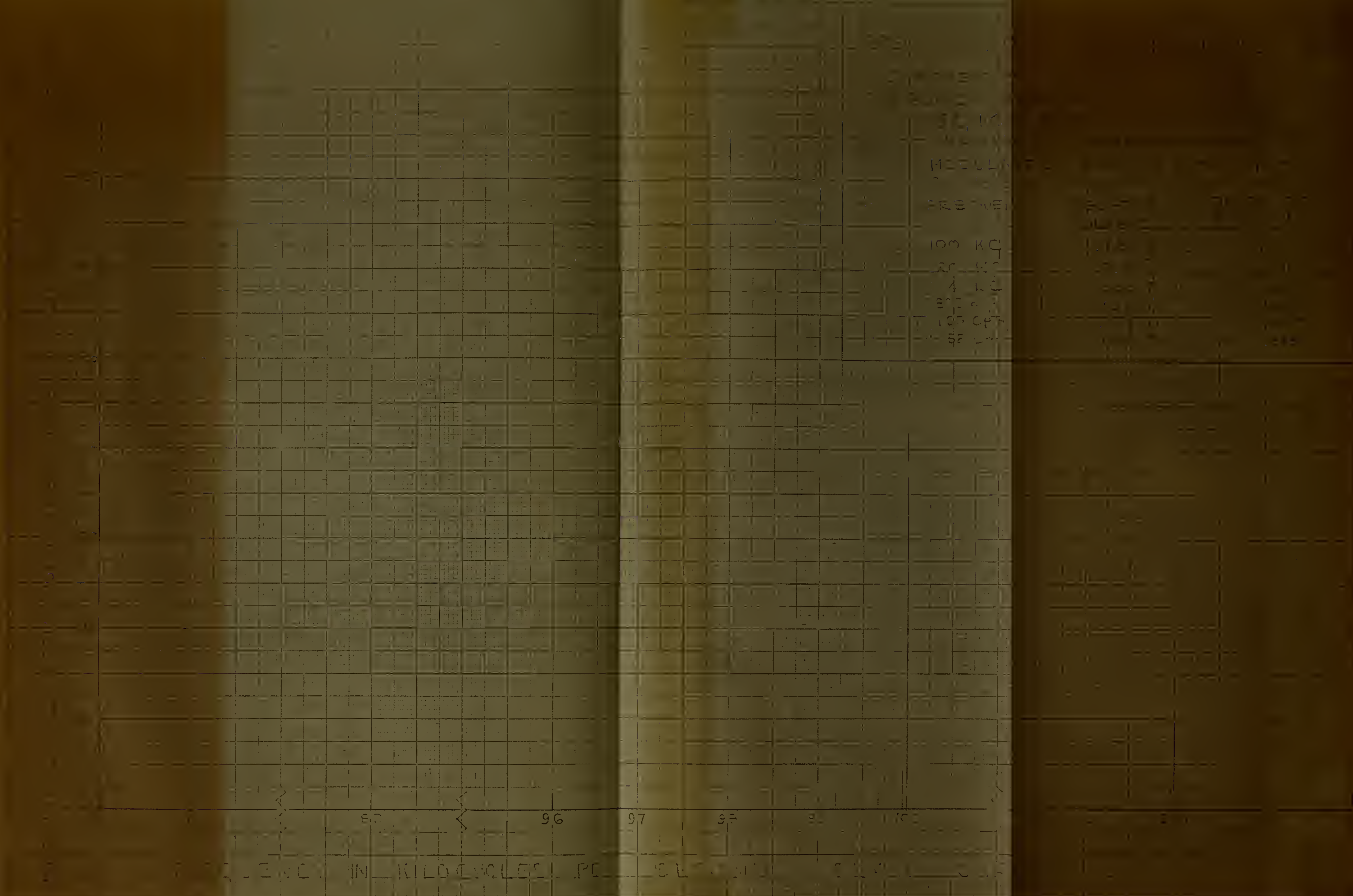

ILLUSTRATION 7




FREQUENCY IN KILOCYCLES PER SECOND



CUMULATIVE POWER VS BANDWIDTH
FM CARRIER ; $\Phi_v = 1.5$
6 TONES OF EQUAL AMPLITUDE
PERCENTAGE OF TOTAL POWER
100
80
60
40
20
±100K
±200K
±300K
±400K
±500K
±600K
±700K
±800K
BANDWIDTH IN CYCLES


100 KC
96
97
IN KILOCYCLES
10

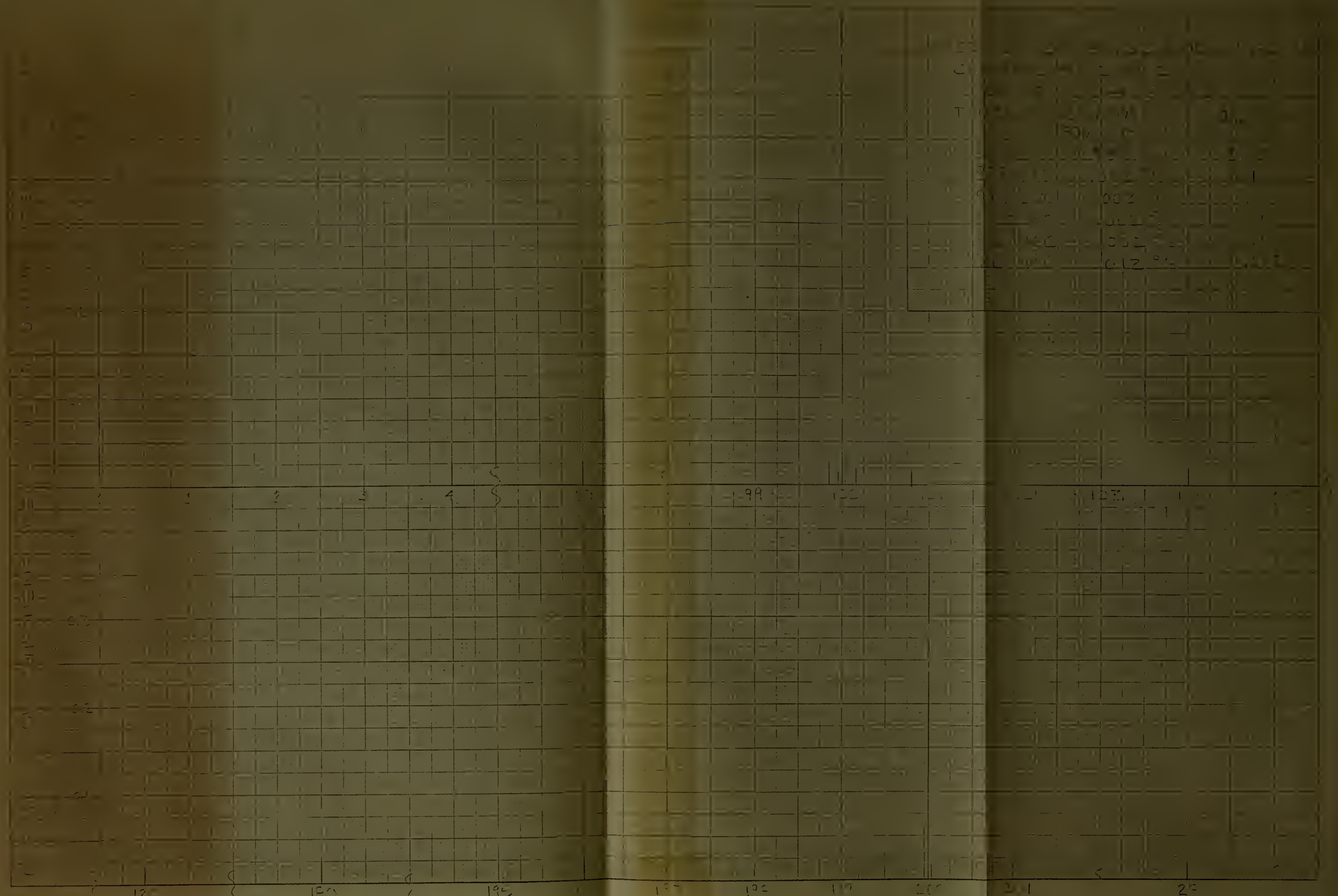


ILLUSTRATION 1 FREQUENCY IN ... PER SECOND ...